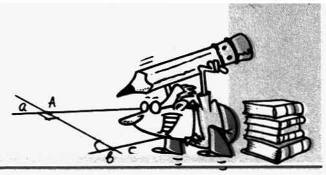


📖 From the school book

Exercise 5 — On factorizing the sum and difference of two cubes

1 Factorize each of the following :

1 📖 $x^3 + 8$
2 $x^3 - 1$
3 $64x^3 + 27$
4 📖 $8x^3 - 125$
5 $125 + a^3$
6 $343 - 27m^3$
7 📖 $m^3 + 64n^3$
8 📖 $512x^3 - y^3$
9 $x^3y^3 + 27$
10 $27x^3y^3 - 64$
11 📖 $\frac{1}{8}a^3 - 8b^3$
12 $\ell^3 - \frac{1}{125}$
13 $8a^3 + 0.001$
14 📖 $0.027m^3 - n^3$
15 $1 + 125b^6$
16 📖 $8x^3 - 343y^6$
17 $x^6 + y^6$
18 $x^6 - 64$

2 Factorize each of the following perfectly :

1 $2x^3 + 16$
2 $3x^3 - 81$
3 $\ell^4 + 64\ell$
4 📖 $\ell^3 m - 27m^4$
5 $3x^4 + 3x$
6 $2x^5 - 54x^2$
7 $16x^3 + 250y^3$
8 📖 $16a^3b + 686b^4$
9 $54x^4y^2 - 16xy^5$
10 $500x^8y^2 - 256x^5y^5$
11 $\frac{1}{2}x^3 + 4$
12 $\frac{1}{3}x^3 - 9$

3 Choose the correct answer from those given :

1 📖 If $x + y = 3$, $x^2 - xy + y^2 = 5$, then $x^3 + y^3 =$ ....
(a) 15 (b) 25 (c) 8 (d) 7

2 If $x^3 - y^3 = 14$, $x^2 + xy + y^2 = 7$, then $x - y =$ ....
(a) 2 (b) 7 (c) 14 (d) -2

3 If $x^3 + y^3 = 28$, $x + y = 2$, then $x^2 - xy + y^2 =$ ....
(a) 28 (b) 14 (c) 2 (d) 7

4 If $y^3 - a = (y - 2)(y^2 + 2y + 4)$, then a = ...
(a) 2 (b) 4 (c) 8 (d) -8

5 If $x^3 - 8 = (x + a)(x^2 + 2x + 4)$, then a = ...
(a) 4 (b) -4 (c) 2 (d) -2

6 If $x^3 + 27 = (x + 3)(x^2 + k + 9)$, then "k" equals ....
(a) $-6x$ (b) $-3x$ (c) $3x$ (d) $6x$

7 $x^3 - k^3 = (x - k)(x^2 + 4x + k^2)$, then k = ...
(a) 2 (b) 4 (c) 16 (d) 64

8 $(x - y)(x + y)(x^4 + x^2y^2 + y^4) =$ ....
(a) $x^3 - y^3$ (b) $x^3 + y^3$ (c) $x^6 - y^6$ (d) $x^6 + y^6$

4 Complete the following to get true statement :

1. $x^3 - 1 = (x - 1)(\cdots\cdots\cdots\cdots\cdots)$
2. $8a^3 + 125 = (\cdots\cdots + \cdots\cdots)(4a^2 - 10a + \cdots\cdots)$
3. $x^{12} + y^{15} = (\cdots\cdots + \cdots\cdots)(\cdots\cdots - \cdots\cdots + \cdots\cdots)$
4. $8a^3 - \cdots\cdots = (\cdots\cdots - \cdots\cdots)(\cdots\cdots + \cdots\cdots + 9)$
5. If $x - 3$ is a factor of the expression $x^3 - 27$, then the second factor is ………
6. If $4a^2 - 2a + 1$ is a factor of the expression $8a^3 + 1$, then the other factor is ………

5 If $x^2 - y^2 = 20$, $x - y = 2$, $x^2 - xy + y^2 = 28$

Find the value of $x^3 + y^3$

6 Factorize each of the following :

1. $(a + b)^3 - b^3$
2. $(x + 5)^3 - 125$
3. $(m - 2n)^3 - 8n^3$
4. $8(m + n)^3 - n^3$
5. $2 - 2(x - 1)^3$
6. $(x + 5)^3 + (x - 5)^3$
7. $(x + y)^3 - (x - y)^3$
8. $(m - n) + (m - n)^4$
9. $(x^3 - 2)(x^3 + 2) - 4$
10. $(x - 3)(x^2 + 3x + 9) + 28$

7 Factorize each of the following :

1. $m^6 - 3m^3 + 2$
2. $y^6 + 26y^3 - 27$
3. $x^6 - 28x^3 + 27$
4. $x^6 - 7x^3 - 8$

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8 Factorize perfectly : $(x + 5)^4 - x - 5$

9 If $xy = 2$, $x - y = 1$, then find the value of $x^3 - y^3$

10 Two integers whose sum is 2 and the sum of their squares = 34 Find the sum of their cubes.



From the school book

Exercise 6 On factorizing by grouping

1 Factorize each of the following perfectly :

1. $a x + b x + a y + by$
2. $ab - bd + ah - dh$
3. $a x + y x + y + a$
4. $am - an + m - n$
5. $a x - c y - c x + a y$
6. $m x - m y - n x + n y$
7. $x y + 5 y + 7 x + 35$
8. $7 x - 28 + a x - 4 a$
9. $5 \ell - 10 m - a \ell + 2 am$
10. $3 a x - a - 6 b x + 2 b$

2 Factorize each of the following perfectly :

1. $c^2 + cd + dh + ch$
2. $6 m^2 - n + 2 m - 3 mn$
3. $8 mn - 2 m^2 + 12 n \ell - 3 m \ell$
4. $x^2 - 2 x z - 2 x y + 4 y z$
5. $a^2 + 2 ab + b^2 - c^2$
6. $25 x^2 - 10 x + 1 - y^2$
7. $1 - x^2 - 4 x y - 4 y^2$
8. $x^2 - y^2 + 4 x + 4 y$
9. $x^2 - 5 x - 4 y^2 + 10 y$
10. $9 x^2 - 4 a^2 + y^2 + 6 x y$
11. $2 x^2 y - x y^2 + 2 a x - a y$
12. $ab x^2 + b x - a x - 1$

3 Factorize each of the following perfectly :

1. $a^3 + a^2 + a + 1$
2. $x^3 - 3 x^2 + 6 x - 18$
3. $a^3 + b^3 - a - b$
4. $x^3 + 2 x^2 - x - 2$
5. $a^3 - 9 a + a^2 - 9$
6. $3 x^3 + 2 x^2 + 12 x + 8$
7. $y^3 + 6 y^2 + 12 y + 8$
8. $a^4 - 3 a^3 - 15 a + 5 a^2$
9. $a^5 - 2 a^2 + a^3 - 2$
10. $x^2 y^3 + 8 x^2 - y^3 - 8$

4 Factorize each of the following perfectly :

1. $x^5 - x^3 - x^2 + 1$
2. $4 m^4 - 9 m^2 + 6 m - 1$
3. $121 x^4 - 100 x^2 - 20 x - 1$

Geometric Applications

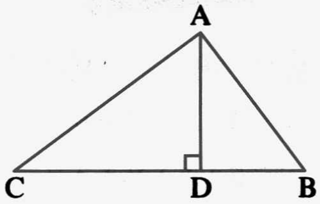
5 A rectangle whose area is $(x^3 + 5 x^2 + 3 x + 15)$ cm^2 and width $(x + 5)$ cm. , find its length in terms of x , then find its perimeter when $x = 2$ cm.

6 **In the opposite figure :**

ABC is a triangle of area $\frac{1}{2}(x^3 + 2x + 2x^2 + 4)$ cm^2.

If its altitude AD is $(x + 2)$ cm.

, what is the length of its base $\overline{BC}$?



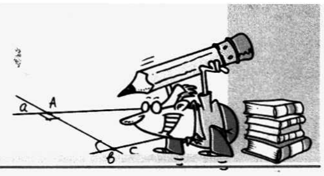
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7 **Factorize each of the following perfectly :**

1. $2x^3(x + 3) - 18x^2 - 54x$
2. $a(a + 4b) + 4b^2 - 9$
3. $a^3 - 4b^3 + ab(a - 4b)$
4. $a^2(b - 5) - 7a(b - 5) - 18b + 90$

8 **Factorize each of the following perfectly :**

1. 📖 $x^2 - 4xy + x - 2y + 4y^2$
2. $3x^2 - 15x - 72 - xy + 8y$
3. $a^3 + a - 2$
4. $a^3 + a^2 + 4$



📖 From the school book

Exercise 7 On factorizing by completing the square

1 Factorize each of the following perfectly :

1. $x^4 + 4$
2. $x^4 + 64$
3. 📖 $x^4 + 4y^4$
4. $x^4 + 64y^4$
5. 📖 $a^4 + 2500b^4$
6. 📖 $81x^4 + 4z^4$
7. 📖 $4x^4 + 625z^4$
8. $64x^4 + 81y^4$
9. $12x^4 + 3y^4$
10. 📖 $8x^4y^2 + 162z^4y^2$

2 Factorize each of the following completely :

1. $9x^4 + 2x^2 + 1$
2. $x^4 - 28x^2 + 16$
3. 📖 $x^4 + 9x^2 + 81$
4. 📖 $9x^4 - 25x^2 + 16$
5. $x^4 + 3x^2y^2 + 4y^4$
6. 📖 $m^4 - 11m^2n^2 + n^4$
7. 📖 $x^4 + x^2y^2 + 25y^4$
8. 📖 $a^4 + 4a^2b^2 + 16b^4$
9. $x^4 + y^4 - 7x^2y^2$
10. 📖 $16x^4 - 28x^2y^2 + 9y^4$
11. 📖 $4x^4 + 25y^4 - 29x^2y^2$
12. 📖 $3m^4 + 3n^4 - 54m^2n^2$
13. $50x^4 + 18y^4 - 68x^2y^2$
14. $18ab^4 - 114b^2c^2a + 128ac^4$

3 Factorize each of the following completely :

1. $x^2(9x^2 - 10y^2) + y^4$
2. 📖 $x^2(x^2 - 19y^2) + 25y^4$
3. 📖 $4x^2(4x^2 - 7y^2) + y^4$
4. 📖 $4a^2(a^2 - 6b^2) + 9b^4$

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4 Factorize each of the following completely :

1. 📖 $x^8 - 16y^8$
2. $x^8 - 21x^4 - 100$
3. $x^8 - 5x^4y^4 - 36y^8$
4. $81x^8 - 17x^4y^4 - 64y^8$

General exercise on factorizing the algebraic expressions

Solve each of the following perfectly :

1 $25x^2 - 9y^2$

2 $2x^5 + 54x^2$

3 $2y^2 + 5y + 3$

4 $2x^4 - 18$

5 $2x^2 - 20x + 48$

6 $x^2 + 8x + 16$

7 $8x^3 + 27$

8 $y^2 - 50y - 51$

9 $25x^2 - 30x + 9$

10 $x^2 - 81$

11 $y^5 - y$

12 $3x^2 + 7x - 6$

13 $x^2 - 8x + 12$

14 $3x^3 + 2x^2 + 12x + 8$

15 $x^3 - 125$

16 $4x^2 - 12x + 9$

17 $a^3 + 3a^2 - 9a - 27$

18 $(x+2)^3 - 4x - 8$

19 $-2x^2 - 15x - 7$

20 $x^2 - 7x + 10$

21 $4x^4 + y^4$

22 $9x^4 - 16y^4$

23 $x^4 - 9x^2 + 20$

24 $1 - 4x^2$

25 $a^6 - 625b^6$

26 $(x+y)^3 - x^3$

27 $49x^2 + 70xy^2 + 25y^4$

28 $5x^2 - 3x - 2$

29 $x^4 - 11x^2y^2 + y^4$

30 $3x^4 - 15x^3 + 12x^2$

31 $3x^2 - 19x + 6$

32 $4x^2 + 28xy + 49y^2$

33 $x^6 - 64y^6$

34 $2y^4 - 4y^3 + 7y - 14$

35 $15a^4 - 21b^2 - 6a^2b$

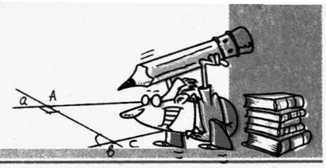
36 $6x^2 + y(2y - 7x)$

37 $64x^4 + y^4$

38 $x^4 - 5x^2 - 24$

39 $20x^4 + 40x^2y^2 + 45y^4$

40 $9x^4 - 13x^2y^2 + 4y^4$



📖 From the school book

Exercise 8 On solving quadratic equations in one variable algebraically

1 **Find in $\mathbb{R}$ the S.S. of each of the following equations :**

1. 📖 $x^2 - 6x = 0$
2. $x^2 - 16 = 0$
3. $4x^2 - 25 = 0$
4. $x^2 + 5x + 6 = 0$
5. 📖 $x^2 - 8x + 15 = 0$
6. $x^2 + 7x - 18 = 0$
7. $x^2 - x - 20 = 0$
8. 📖 $6x^2 - 7x - 3 = 0$
9. $2x^2 + 7x - 4 = 0$
10. $2x^2 - 5x - 3 = 0$
11. $x^2 + 4x + 4 = 0$
12. $9x^2 - 6x + 1 = 0$

2 **Find in $\mathbb{R}$ the S.S. of the following equations :**

1. $x^2 = x$
2. $3x^2 = 7x$
3. $4x^2 = 49$
4. 📖 $x^2 + x = 6$
5. $x^2 - 8x = -15$
6. $x^2 - 15 = 2x$
7. $2x^2 - 10x = -12$
8. 📖 $6x^2 - x = 22$
9. 📖 $5x^2 + 12x = 44$
10. 📖 $12x^2 = 47x - 45$
11. $5(x^2 + 3) = 60$
12. $x(x - 3) = 5x$

3 **Find in $\mathbb{R}$ the S.S. of each of the following equations :**

1. $x(x - 5) + 6 = 0$
2. $x(x + 3) = 10$
3. $(x + 4)(x - 2) + 5 = 0$
4. 📖 $(x - 3)(x + 1) = 5$
5. $(x + 8)(x - 3) = 3x$
6. $2x(x - 5) - 4(5 - x) = 0$
7. 📖 $(x + 3)^2 - 49 = 0$
8. $(x - 2)^2 = 81$
9. $4(x + 5)^2 = 25$
10. 📖 $(x - 1)^2 + x = 3$
11. $2(x + 3)^2 + 7(x + 3) = 0$
12. $(2x + 1)^2 = (3x - 1)^2$
13. 📖 $(2x - 1)^2 + (x - 1)^2 = 10$
14. 📖 $(x + 3)^2 + 3(x + 3) - 10 = 0$

4 **Find in $\mathbb{R}$ the S.S. of each of the following equations :**

1. $2x^3 - 8x = 0$
2. 📖 $4x^3 = 9x$
3. 📖 $x^4 - 5x^2 + 4 = 0$
4. $x^4 - 26x^2 = -25$
5. $x^4 - 16 = 0$

5 **Find in $\mathbb{R}$ the S.S. of each of the following equations :**

1. $y^2 - \frac{7y}{3} = -\frac{4}{3}$
2. $x^2 - \frac{2x + 3}{2} = \frac{9}{2}$
3. $x + \frac{2}{x} = 3$
4. $x - \frac{5}{x} = \frac{1}{2}$
5. $\frac{x - 1}{5} = \frac{6}{x}$

6 **Choose the correct answer from those given :**

1. The S.S. of the equation $x(x - 2) = 0$ in $\mathbb{R}$ is ………

(a) $\{0\}$ (b) $\{0, -2\}$ (c) $\{0, 2\}$ (d) $\{2\}$

2 The S.S. of the equation : $3(x-2)(x+5)=0$ in $\mathbb{R}$ is ....

(a) $\{0, 2, -5\}$ (b) $\{3, 2, -5\}$ (c) $\{2, -5\}$ (d) $\{-2, 5\}$

3 The S.S. of the equation : $x^2-4=0$ in $\mathbb{R}$ is ....

(a) $\{4\}$ (b) $\{4, -4\}$ (c) $\{2\}$ (d) $\{2, -2\}$

4 The S.S. of the equation : $x^2+25=0$ in $\mathbb{R}$ is ....

(a) $\{5\}$ (b) $\{5, -5\}$ (c) $\{-5\}$ (d) $\varnothing$

5 The S.S. of the equation : $(x-4)^2=0$ in $\mathbb{R}$ is ....

(a) $\{4\}$ (b) $\{0, 4\}$ (c) $\{0, -4\}$ (d) $\{-4\}$

6 The S.S. of the equation : $x(x-3)=5x$ in $\mathbb{R}$ is ....

(a) $\{3\}$ (b) $\{0, 3, 5\}$ (c) $\{3, 5\}$ (d) $\{0, 8\}$

7 The S.S. of the equation : $\frac{4}{x}=\frac{x}{9}$ in $\mathbb{R}$ is ....

(a) $\{4, 9\}$ (b) $\{6, -6\}$ (c) $\{6\}$ (d) $\{36\}$

8 The equation whose roots are 3 and 5 is ....

(a) $5x^2+8x+3=0$ (b) $2x^2+8x-15=0$

(c) $x^2-8x+15=0$ (d) $3x^2+8x+5=0$

---

7 **Complete the following :**

1 If -5 is a root of the equation : $x^2+2x-15=0$

, then the other root is ....

2 If $x=2$ is a root of the equation : $x^2-6x+k=0$, then k = ...

and the other root is ....

3 If one of the roots of the equation : $2x^2+8x=0$

is a root of the equation : $x^2+5x+a=0$, then a = ... or ....

4 The S.S. of the equation : $x-\frac{2}{x}=\frac{7}{2}$ is ....

---

8 **If :** $x+\frac{1}{x}=2$ **, then find :** $x^2+\frac{1}{x^2}$

For excellent pupils

9 **If :** $x^2+\frac{1}{x^2}=34$ **, then find :** $x+\frac{1}{x}$

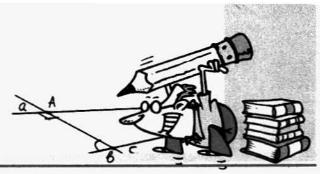
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10 **Find in $\mathbb{R}$ the S.S. of the equation :** $\frac{x(x-2)}{6}-\frac{x(x+1)}{4}+\frac{7(x-3)}{3}-2=0$

---

11 If 2 and 3 are two solutions of the equation : $x^2-bx+c=0$,

find the value of b and c



From the school book

Exercise 9 Applications on solving quadratic equations in one variable algebraically

1 **Choose the correct answer from those given :**

1. If the age of Bassim now is x years , then his age 3 years ago was .... years.
   (a) $3x$ (b) $x+3$ (c) $x-3$ (d) x^3
2. If the age of Amgad now is x years , then his age after 7 years will be .... years.
   (a) $7x$ (b) $x-7$ (c) $x+7$ (d) x^7
3. If the age of Ayman 5 years ago was x years , then his age now is .... years.
   (a) $x-5$ (b) $x+5$ (c) $5x$ (d) $\frac{x}{5}$
4. If the age of Sally 2 years ago was x years , then her age after 3 years from now will be .... years.
   (a) $x+2$ (b) $x+3$ (c) $x+5$ (d) $6x$
5. If the age of Magdy now is x years , then the square of his age after 2 years is ....
   (a) x^2+2 (b) x^2+4 (c) $(x-2)^2$ (d) $(x+2)^2$
6. If the age of Samy now is x years , then twice his age 5 years ago is .... years.
   (a) $x-5$ (b) $2x-5$ (c) $x-10$ (d) $2x-10$
7. Three times the square of the number x is ....
   (a) $(3x)^2$ (b) x^2+3 (c) $3x^2$ (d) $\frac{x^2}{3}$

---

2 A positive integer whose square is more than five times the number by 36

Find the number. « 9 »

---

3 An integer , if we add twice its square to the number 7 the result will be 135

Find the number. « 8 or – 8 »

---

4 Find the rational number whose four times its square equals 81 « $\frac{-9}{2}$ or $\frac{9}{2}$ »

---

5 A positive integer whose square equals six times the number. Find the number. « 6 »

---

6 📖 What is the real number if it is added to its square , the result will be 12 ? « 3 or – 4 »

---

7 Find the positive rational number whose square is more than its twice by 48 « 8 »

---

8 Divide the number 20 into two numbers whose product is 75 « 15 , 5 »

9 Two real numbers , the difference between them is 5 and the sum of their squares is 73

Find the two numbers. « 3 , 8 or – 3 , – 8 »

---

10 📖 Find two real numbers whose product is 45 and one of them is 4 more than the other.

« – 9 , – 5 or 5 , 9 »

---

11 📖 The sum of the squares of two successive odd numbers is 130

Find the two numbers. « – 9 , – 7 or 7 , 9 »

---

12 📖 The sum of three successive integers is equal to the square of their middle integer.

Find these numbers. « 2 , 3 and 4 or – 1 , 0 and 1 »

---

13 Two integers , the ratio between them is 7 : 8 and their product is more than nine times the greater number by 80

Find the two numbers. « 14 , 16 »

---

14 A positive integer , if we add twice its square to its additive inverse the result will be 91

Find this number. « 7 »

---

15 📖 What is the real number which exceeds its multiplicative inverse by $\frac{5}{6}$? « $\frac{3}{2}$ or $\frac{-2}{3}$ »

---

16 A number is formed from two digits , its units digit is twice the tens digit and the product of the two digits exceeds their sum by 9

Find the number. « 36 »

Life Applications

17 The square of age of Said now is more than three times his age four years ago by 192

Find his age now. « 15 years »

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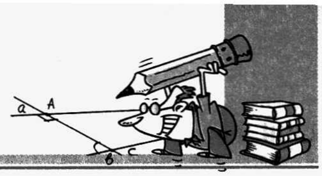
18 📖 Hatem is 4 years older than Hanan now , and the sum of squares of their ages now is 26

Find their ages now. « 5 years , one year »

---

19 If the age of Kamal now is more than the age of his brother Anees by 3 years and 4 years ago the product of their ages was 18

Find the age of each of them now. « 7 years , 10 year »



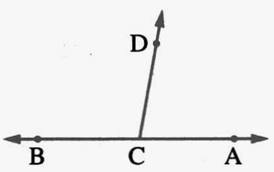
Geometric Applications

20 Find the dimensions of a rectangle whose length is 4 cm. more than its width and whose area is 21 cm^2 « 3 cm. , 7 cm. »

21 A rectangle whose area is 46 cm^2 and its length is 7.5 cm. more than its width. Find its perimeter. « 31 cm. »

22 A rectangle whose length is more than its width by 5 cm. If its area is less than the area of a square whose side length is three times the width of the rectangle by 57 cm^2
Find the two dimensions of the rectangle and the side length of the square.
« 3 cm. , 8 cm. , 9 cm. »

23 **In the opposite figure :**



$\overrightarrow{CD} \cap \overleftrightarrow{AB} = \{C\}$ If m ($\angle$ BCD) = $(x^2)^\circ$,
m ($\angle$ ACD) = $8x^\circ$
Calculate the value of x « 10° »

24 In the triangle ABC : m ($\angle$ A) = $(x^2 + 61)^\circ$,
m ($\angle$ B) = $(110 - 11x)^\circ$ and m ($\angle$ C) = $(90 - 7x)^\circ$
Find the value of x and the measures of all angles. « 9° , 142° , 11° , 27° »

25 A right-angled triangle , the length of one side of the right angle is more than the length of the other side of the right angle by 2 cm. and its area = 24 cm^2
Find the lengths of the sides of the right angle. « 8 cm. , 6 cm. »

26 A right-angled triangle whose two right angle sides lengths are $(5x + 3)$ cm. and $(x + 5)$ cm. and its area is 24 cm^2 Calculate its perimeter. « 24 cm. »

27 A right-angled triangle whose sides lengths are $(2x)$ cm. , $(2x + 1)$ cm. and $(x - 11)$ cm.
Find the value of x and calculate the perimeter and the area of the triangle.
« 20 , 90 cm. , 180 cm^2 »

28 A rectangle whose length is twice its width , if its length increases by 1 cm. and its width decreases by 1 cm. , then its area decreases by 7 cm^2
Find the length and the width of the rectangle. « 6 cm. , 12 cm. »

For excellent pupils

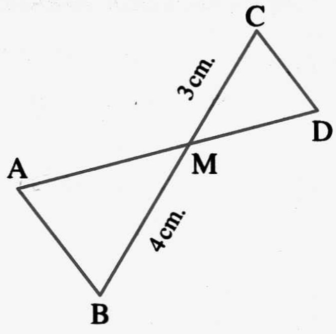
29 **In the opposite figure :**

Δ MCD ~ Δ MAB ,

MB = 4 cm. , MC = 3 cm. ,

AD = 7 cm. , MA > MC

Find the length of $\overline{MA}$

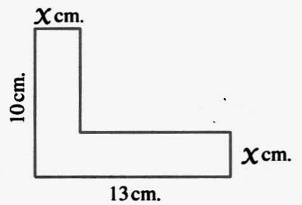


« 4 cm. »

---

30 If the area of the opposite figure = 60 cm^2 ,

find the value of x



« 3 cm. »

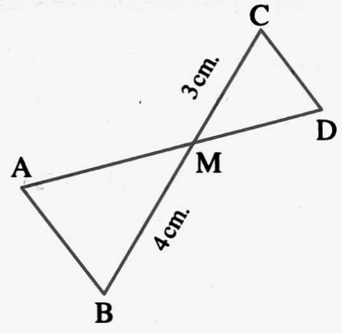
---

31 A room whose width is 9 metres and its length is 12 metres.
A decoraster wanted to buy a carpet for the room in condition that he left around the carpet a rectangular tape of a fixed width uncovered. Calculate the width of the tape if the carpet covers half the area of the room. « 1.5 m. »



For excellent pupils

29 **In the opposite figure :**



Δ MCD ~ Δ MAB ,

MB = 4 cm. , MC = 3 cm. ,

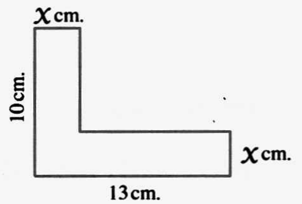
AD = 7 cm. , MA > MC

Find the length of $\overline{MA}$ « 4 cm. »

---

30 If the area of the opposite figure = 60 cm^2 ,

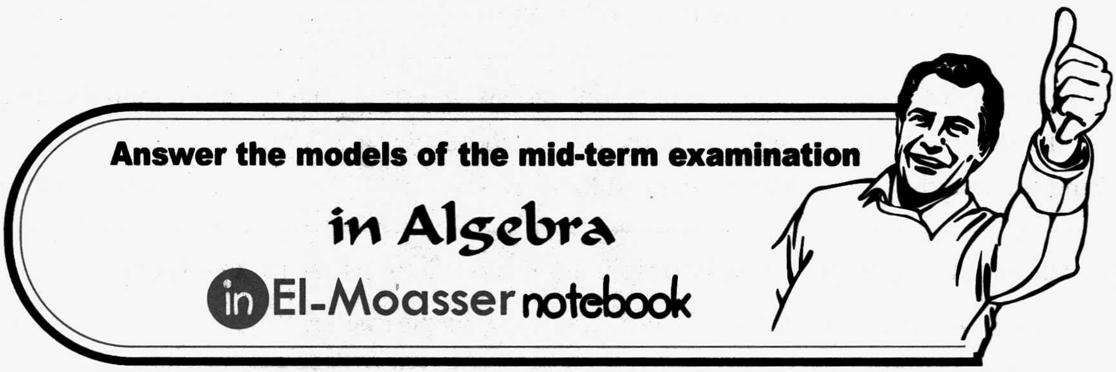
find the value of x



« 3 cm. »

---

31 A room whose width is 9 metres and its length is 12 metres.
A decoraster wanted to buy a carpet for the room in condition that he left around the carpet a rectangular tape of a fixed width uncovered. Calculate the width of the tape if the carpet covers half the area of the room. « 1.5 m. »





📖 From the school book

Exercise 10 On non-negative and negative integer powers in $\mathbb{R}$

1 **Find the value of each of the following in the simplest from :**

1 3^{-2}	2 📖 $\left(\frac{1}{4}\right)^{-1}$	3 $\left(\frac{2}{3}\right)^{-2}$
4 📖 $\left(\sqrt{5}\right)^{4}$	5 $\left(\sqrt{3}\right)^{-2}$	6 📖 $\left(-\sqrt{3}\right)^{-2}$
7 $\left(\sqrt[3]{5}\right)^{-3}$	8 $\frac{1}{\left(\sqrt{5}\right)^{-2}}$	9 📖 $(0.01)^{-2}$
10 $(0.2)^{-2}$	11 $\left(\sqrt{2}\right)^{-3}$	12 📖 $\left(\frac{\sqrt{3}}{3}\right)^{-5}$

2 **Simplify each of the following to the simplest form where $x \neq 0$:**

1 $x^{3} \times x^{-2} \times x^{-1}$	2 $x^{-4} \div x^{-3}$	3 $\left(x^{2}\right)^{-3} \times \left(x^{-3}\right)^{-2}$
4 $\frac{x^{2} \times x^{-3}}{x^{-4} \times x}$	5 $\frac{\left(x^{2}\right)^{-3} \times \left(x^{-1}\right)^{2}}{x^{-3} \times x^{-4}}$	

3 **Simplify each of the following to the simplest form :**

1 📖 $\left(\sqrt{2}\right)^{2} \times \left(\sqrt{2}\right)^{4}$	« 8 »	2 $\left(\sqrt{7}\right)^{5} \times \left(\sqrt{7}\right)^{-2} \times \left(\sqrt{3}\right)^{-1}$	« 7 »
3 $\left(\sqrt{2}\right)^{4} \times \left(-\sqrt{2}\right)^{2} \times \left(\sqrt{2}\right)^{-2}$	« 4 »	4 $\sqrt{3} \times \left(-\sqrt{3}\right)^{3} \times \left(-\sqrt{3}\right)^{4}$	« – 81 »
5 $\left(\sqrt{5}\right)^{-4} \div \left(\sqrt{5}\right)^{-6}$	« 5 »	6 📖 $\left(-\sqrt{5}\right)^{9} \div \left(-\sqrt{5}\right)^{5}$	« 25 »
7 📖 $\left(\frac{-1}{\sqrt{2}}\right)^{6}$	« $\frac{1}{8}$ »	8 📖 $\left(\left(\sqrt{2}\right)^{3} \times \left(-\sqrt{2}\right)^{2}\right)^{2}$	« 32 »
9 📖 $\left(\sqrt{3}\right)^{-4} \times \left(-\sqrt{2}\right)^{4}$	« $\frac{4}{9}$ »	10 📖 $\left(\left(-5\right)^{3}\right)^{2} \times \left(-\sqrt{5}\right)^{-4}$	« 625 »

4 **Simplify each of the following to the simplest form :**

1 $\frac{\left(\sqrt{7}\right)^{-4} \times \left(\sqrt{7}\right)^{-3}}{\left(\sqrt{7}\right)^{-9}}$	« 7 »	2 📖 $\frac{\left(\sqrt{3}\right)^{7} \times \left(\sqrt{3}\right)^{8}}{\left(\sqrt{3}\right)^{6}}$	« $81\sqrt{3}$ »
3 $\frac{\left(\sqrt{3}\right)^{8} \times \left(-\sqrt{3}\right)^{6}}{\left(\sqrt{3}\right)^{12}}$	« 3 »	4 $\frac{\left(\sqrt{5}\right)^{10} \times \left(-\sqrt{5}\right)^{5}}{\left(\sqrt{5}\right)^{11}}$	« – 25 »

5 $\dfrac{(2\sqrt{7})^{-2} \times (\sqrt{2})^{-4}}{(\sqrt{7})^{-2}}$ « $\frac{1}{16}$ »

6 $\dfrac{(3\sqrt{2})^{4} \times (\sqrt{2})^{2}}{(2\sqrt{3})^{2}}$ « 54 »

7 $\dfrac{(\sqrt{3})^{-4} \times (\sqrt{2})^{3} \times (3\sqrt{3})^{5}}{(3\sqrt{2})^{5} \times \sqrt{3}}$ « $\frac{1}{2}$ »

8 📖 $\dfrac{(\sqrt{3})^{5} \times (\sqrt{3})^{4}}{(\sqrt{3})^{3} \times 27}$ « 1 »

9 $\dfrac{(\sqrt{2})^{5} \times (10)^{6}}{(\sqrt{2})^{3} \times 2^{3} \times 5^{5}}$ « 80 »

10 📖 $\dfrac{(15)^{-2} \times (\sqrt{5})^{3} \times 3^{3}}{9 \times (\sqrt{5})^{-3}}$ « $\frac{5}{3}$ »

11 $\dfrac{(\sqrt{5})^{6} \times (\sqrt{2})^{8}}{(\sqrt{10})^{4}}$ « 20 »

12 📖 $\dfrac{(10)^{2} \times (10)^{-7}}{(0.1)^{2} \times 0.001}$ « 1 »

5 Simplify each of the following to the simplest form :

1 $\left(\dfrac{2\sqrt{3}}{\sqrt{2}}\right)^{4}$ « 36 »

2 📖 $\left(\dfrac{3\sqrt{2}}{2\sqrt{3}}\right)^{4}$ « $\frac{9}{4}$ »

3 $\left(\dfrac{-5\sqrt{2}}{2\sqrt{5}}\right)^{4}$ « $\frac{25}{4}$ »

4 $\left(\dfrac{\sqrt{3}}{\sqrt{2}}\right)^{2} \times \left(\dfrac{\sqrt{2}}{\sqrt{3}}\right)^{4}$ « $\frac{2}{3}$ »

6 Simplify to the simplest form :

1 $\dfrac{9^{x} \times 3^{x+2}}{(27)^{x}}$ « 9 »

2 $\dfrac{2^{2x} \times 3^{x-1}}{(12)^{x}}$ « $\frac{1}{3}$ »

3 $\dfrac{2^{x} \times 4^{x+1}}{8^{x}}$ « 4 »

4 $\dfrac{(36)^{n} \times 5^{2n}}{(30)^{2n}}$ « 1 »

5 $\dfrac{2^{x} \times (49)^{x-1}}{(98)^{x}}$ « $\frac{1}{49}$ »

6 $\dfrac{4^{x+2} \times 9^{3+x}}{6^{2x+3}}$ « 54 »

7 $\dfrac{4^{n} \times 6^{2n}}{2^{4n} \times 3^{2n}}$ « 1 »

8 $\dfrac{(81)^{x} \times 6^{2x}}{(27)^{2x-1} \times 4^{x}}$ « 27 »

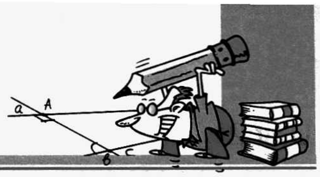
9 $\dfrac{2^{n} \times 9^{n+1} \times (\sqrt{2})^{2}}{6 \times (18)^{n}}$ « 3 »

10 $\dfrac{3 \times (18)^{x+1} \times 2^{x}}{2 \times (36)^{x}}$ « 27 »

11 $\dfrac{6^{n} \times 4^{n+\frac{1}{2}}}{(24)^{n}}$ « 2 »

12 📖 $\dfrac{8^{n-1} \times 32^{-n}}{32 \times 4^{-n}}$ « $\frac{1}{256}$ »

13 $\dfrac{(\sqrt{3})^{n+2} \times (15)^{n+1}}{(\sqrt{3})^{n} \times 3^{n} \times 5^{n+2}}$ « $\frac{9}{5}$ »



14 📖 $\frac{4^{x+1} \times 9^{2-x}}{6^{2x}}$, then find the value of the result , when $x = 1$ « $2^2 \times 3^{4-4x}$, 4 »

15 $\frac{9^{x-1} \times (\sqrt{2})^{6x}}{8^{x} \times (\sqrt{3})^{2x}}$, then find the value of the result , when $x = 2$ « 1 »

16 $4^{x-1} \times 2^{3x+2} \times \left(\frac{1}{2}\right)^{3x}$, what is the value of the result if $2^{x} = 5$? « 25 »

---

7 Prove that :

1 $\frac{(\sqrt{2})^2 \times 2^{1-x} \times 12^{2x-1}}{8^{x} \times 9^{x}} = \frac{1}{3}$

2 $\frac{(27)^{x-1} \times 8^{x}}{(2\sqrt{2})^{2x} \times (3\sqrt{3})^{2x}} = \frac{1}{27}$

---

8 📖 If $\frac{8^{x} \times 9^{x}}{18^{x}} = 64$, find the value of 4^{-x} « $\frac{1}{64}$ »

---

9 📖 If $a = \sqrt{3}$ and $b = \sqrt{2}$, find the value of :

1 $a^4 - b^4$

2 $\frac{a^4}{b^4}$ « 5 , $\frac{9}{4}$ »

---

10 📖 If $x = 2\sqrt{2}$ and $y = 3$, find the value of : $(x^2 - y^2)^3$ « – 1 »

---

11 If $a = \frac{3}{\sqrt{2}}$ and $b = \frac{\sqrt{2}}{\sqrt{3}}$, find the value of : $2\,(ab)^2 \times \left(\frac{b}{a}\right)^{-2}$ « $\frac{81}{2}$ »

---

12 📖 If $x = \frac{\sqrt{3}}{2}$, $y = \frac{1}{\sqrt{3}}$ and $z = \frac{\sqrt{2}}{2}$, find the value of : $x^2 + (x z)^2 \times y^2$ « $\frac{7}{8}$ »

---

13 If $a = \frac{3\sqrt{2}}{2}$ and $b = \frac{\sqrt{3}}{\sqrt{2}}$, prove that : $\left(\frac{a}{b}\right)^2 - 3\left(\frac{b}{a}\right)^2 = 2$

---

14 📖 If $x = 2$ and $y = \sqrt{3}$, find the value of the following in the simplest form :

1 $3\,(x + y)^4\,(x - y)^4$

2 $\left(\frac{x + y}{x - y}\right)^{-2}$ « 3 , $97 - 56\sqrt{3}$ »

---

15 📖 If $a = \frac{1}{\sqrt{2}}$, $b = -1$, then calculate the value of : $7\,a^6 + (1 - b)^{-3}$ « 1 »

---

16 If $x = 3$, $y = \sqrt{2}$, find in the simplest form the value of each of the following :

1 $x^{-2}\,y^{-4}$ « $\frac{1}{36}$ »

2 $(x^{-2} \times y^4)^{-2}$. « $\frac{81}{16}$ »

3 $\left(\frac{x}{y}\right)^{-3}$ « $\frac{2\sqrt{2}}{27}$ »

17 Choose the correct answer from those given :

1 $5^2 + 5^2 = ...$

(a) 10^2 (b) 10^4 (c) 5^4 (d) 50

2 $3^5 \times 2^5 = ...$

(a) 5^{10} (b) 6^{10} (c) 6^5 (d) 6^{25}

3 $(5\,a)^{zero} = ...$, $a \neq 0$

(a) 5 (b) a (c) 5 a (d) 1

4 $3\,x^{zero} = ...$, $x \neq 0$

(a) zero (b) 1 (c) 3 (d) $3\,x$

5 $3^{(2^3)} = ...$

(a) 3^6 (b) 3^5 (c) 3^8 (d) 3^{32}

6 $(5^2)^3 = ...$

(a) 5^6 (b) 5^5 (c) 5^{32} (d) 5

7 📖 $4^3 + 4^3 + 4^3 + 4^3 = ...$

(a) 4^3 (b) 4^4 (c) 4^{12} (d) 4^{81}

8 $2^5 + 2^5 + 2^5 + 2^5 = ...$

(a) 2^4 (b) 2^6 (c) 2^7 (d) 2^{20}

9 The quarter of the number $4^{20} = ...$

(a) 1^{20} (b) 4^{19} (c) 4^{16} (d) 4^5

10 4 times the number $2^8 = ...$

(a) 2^{32} (b) 8^8 (c) 2^{10} (d) 4^8

11 📖 Sixth the number $2^{12} \times 3^{12}$ is ....

(a) 6^2 (b) 6^4 (c) 6^{11} (d) 6^{23}

12 Fifth the number $\left(\sqrt[3]{5}\right)^6$ is ....

(a) 5 (b) 5^5 (c) 5^6 (d) 5^{12}

13 $\left(\sqrt{3}\right)^6 \times 3^4 = ...$

(a) $\left(\sqrt{3}\right)^{24}$ (b) 3^{10} (c) 3^7 (d) $\left(\sqrt{3}\right)^{10}$

14 📖 The value of : $2^5 + \left(\sqrt{2}\right)^{10} = ...$

(a) 2^6 (b) 2^{10} (c) $\left(\sqrt{2}\right)^{15}$ (d) $\left(\sqrt{2}\right)^{20}$

15 📖 The value of : $2^{20} + 2^{21} = ...$

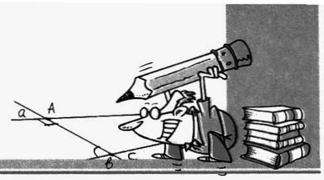
(a) 2×2^{40} (b) 2×2^{41} (c) 3×2^{20} (d) 3×2^{21}

16 📖 What of the following is closest to $11^2 + 9^2$?

(a) 22 + 18 (b) 211 + 29 (c) 120 + 20 (d) 120 + 80

17 If $6^x = 11$, then $6^{x+1} = ...$

(a) 12 (b) 22 (c) 66 (d) 72



18 📖 If $5^{x} = 4$, then $5^{x-1} = \dots\dots$

(a) 1.25 (b) 0.8 (c) 0.125 (d) 0.08

19 📖 $0.002 \times 0.05 = \dots\dots$

(a) 10^{-5} (b) 10^{-4} (c) 10^{4} (d) 10^{5}

20 📖 If $x = \frac{\sqrt{9}}{\sqrt{3}}$, then $x^{-1} = \dots\dots$

(a) $\frac{\sqrt{3}}{3}$ (b) $\frac{\sqrt{3}}{\sqrt{2}}$ (c) $\sqrt{3}$ (d) 2

21 $x^{m-1} \times \dots\dots = 1$, $x \neq 0$

(a) x^{m-1} (b) x^{-m-1} (c) x^{m+1} (d) x^{-m+1}

22 📖 $(\sqrt{3}+\sqrt{2})^{9}(\sqrt{3}-\sqrt{2})^{9} = \dots\dots$

(a) 1 (b) $\sqrt{5}$ (c) $\sqrt{6}$ (d) 5

23 📖 The numerical value of the expression : $\frac{2^{2n+1} \times 5^{2n+1}}{10^{2n}}$ is $\dots\dots$

(a) $\frac{1}{10}$ (b) 7 (c) 10 (d) 100

---

18 Complete the following :

1 $(\sqrt{3})^{4} \times (\sqrt{3})^{2} = 3^{\dots\dots}$

2 $\frac{((\sqrt{7})^{3})^{2}}{((\sqrt{7})^{2})^{3}} = \dots\dots$

3 $((\sqrt{5})^{9})^{11} - ((\sqrt{5})^{11})^{9} = \dots\dots$

4 $a^{\dots\dots} \times a^{6} = a^{6}$

5 $(a^{2} b^{\dots\dots})^{4} = a^{8} b^{12}$

6 📖 The simplest form of the expression : $2^{zero} \times 2^{-1} \times \left(\frac{-1}{\sqrt{2}}\right)^{2} = \dots\dots$

7 The greater number from the two numbers $(-\sqrt{2})^{25}$ and $(-\sqrt{2})^{24}$ is $\dots\dots$

8 If 4 times a number is 4^{2} , then $\frac{3}{4}$ this number is $\dots\dots$

9 If $(x-5)^{zero} = 1$, then : $x \in \dots\dots$

10 If $a = 7^{x}$ and $b = 7^{-x}$, then : $a \times b = \dots\dots$

11 📖 If $x = (\sqrt{2}+3)^{5}$ and $y = (\sqrt{2}+3)^{-5}$, then : $x y = \dots\dots$

12 $\left(\frac{5}{6}\right)^{-4} = \left(-\frac{\dots\dots}{\dots\dots}\right)^{2}$

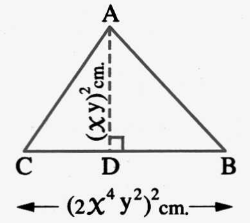
13 If $\left(\frac{1}{2}\right)^{x} = 5$, then : $(8)^{-x} = \dots\dots$

14 If $2^{x} = 7$, $2^{y} = 5$, then : $2^{x+y} = \dots\dots$

15 If $5^{x} = 3$, $5^{-y} = 7$, then : $5^{x+y} = \dots\dots$

Geometric Application

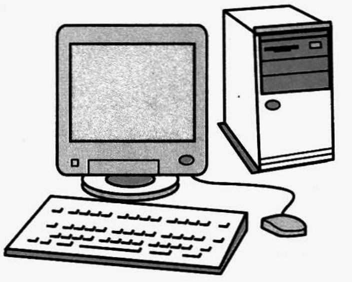
19 Write an algebraic expression in the simplest form that represents the area of the triangle ABC



Life Application

20 A computer with a processor of 5.4 GH can do 5.4×10^9 operations per second , if the number of required operations to convert an MP3 file into an audio file is 2.5×10^{11} operations.

How many seconds does the computer take to convert that file ?



For excellent pupils

21 **Complete the following :**

1. If $x^3 y^{-3} = 8$, then : $y^2 x^{-2} =$ ....
2. If $x = \sqrt{2}$, $y = \left(\sqrt{2}\right)^{-1}$, then : $x^{101} y^{100} =$ ....
3. If $3^{x+2} = 18$, then : $(81)^x =$ ....
4. If $2^x = 3$, $2^y = 5$, then : $4^{2x+y} =$ ....

---

22 **Choose the correct answer from those given :**

1. $5 \times 5 \times 5 \times 2 \times 2 \times 2 \times 2 \times 2 = 4 \times$ ....

   (a) 5^3 (b) 2^3 (c) 10^3 (d) $5^3 + 2^3$

2. $2^{2011} = 2^{2010} +$ ....

   (a) 2 (b) 2010 (c) 2^{2010} (d) 2^{2011}

3. The expression : $2^{1000} + 256^{125} =$ ....

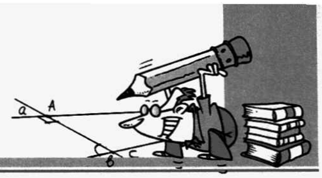
   (a) 258^{125} (b) 258^{1125} (c) 2^{1001} (d) 4^{1000}

4. 📖 If $x \neq 0$ and $x + \frac{1}{x} = \sqrt{5}$, then $x^2 + \frac{1}{x^2} =$ ....

   (a) 1 (b) 3 (c) 5 (d) 7

5. 📖 The digit in the units place of $3^{12} \times 2^{14}$ is ....

   (a) 2 (b) 3 (c) 4 (d) 6



📖 From the school book

Exercise 11 On solving the equations and the exponential equations in $\mathbb{R}$

1 Find the value of n in each of the following when $n \in \mathbb{Z}$:

1. $5^n = 25$ « 2 »
2. $2^{-n} = 32$ « – 5 »
3. $2^{n-1} = 32$ « 6 »
4. 📖 $3^{n-2} = 81$ « 6 »
5. $3^{n-2} = 1$ « 2 »
6. $2^{n-3} = \frac{1}{4}$ « 1 »
7. 📖 $3^{n-2} = \frac{1}{9}$ « 0 »
8. 📖 $\left(\sqrt{3}\right)^{n-1} = 9$ « 5 »
9. 📖 $\left(\frac{2}{5}\right)^{2n-1} = \frac{8}{125}$ « 2 »
10. 📖 $\left(\frac{3}{5}\right)^{n+2} = \frac{125}{27}$ « – 5 »
11. 📖 $\left(\frac{2}{3}\right)^{n-4} = 2\frac{1}{4}$ « 2 »
12. 📖 $\left(\frac{2}{3}\right)^{n+5} = \left(3\frac{3}{8}\right)^{-2}$ « 1 »
13. $5^{2n-4} = 7^{2n-4}$ « 2 »
14. $3^{n-4} = n^{n-4}$ « 3 or 4 »
15. $9 \times 3^{n-4} = 1$ « 2 »
16. $2 \times 4^{n+3} = \frac{1}{32}$ « – 6 »
17. $8 \times \left(\sqrt{2}\right)^{n-3} = 1$ « – 3 »
18. $\frac{1}{27}\left(\sqrt{3}\right)^{n+2} = 1$ « 4 »

2 Find the S.S. of each of the following equations in $\mathbb{R}$:

1. $6^{x^2-4} = 7^{x^2-4}$ « $\{-2, 2\}$ »
2. $2^{x^2-9} = 1$ « $\{-3, 3\}$ »
3. $2^{x^2-x} = 4$ « $\{-1, 2\}$ »
4. $5^{|x|} = 125$ « $\{-3, 3\}$ »
5. 📖 $(32)^{x-3} = 8^{2x+1}$ « $\{-18\}$ »
6. 📖 $3^{x-3} = \left(\sqrt{3}\right)^{x+5}$ « $\{11\}$ »
7. $3^{n-1} \times 2^{1-n} = \frac{27}{8}$ « $\{4\}$ »
8. 📖 $25 \times 3^{x-1} = 9 \times 5^{x-1}$ « $\{3\}$ »

3 Find the value of n in each of the following where $n \in \mathbb{Z}$:

1. $\frac{2^n \times 9^{n+1}}{(18)^n} = 3^n$ « 2 »
2. $\frac{6^{2n-3}}{2^{n-1} \times 3^{n-1}} = 6$ « 3 »
3. $\frac{(12)^{n-1}}{2^{n-1} \times 3^{n-1}} = 1$ « 1 »
4. 📖 $\frac{3^n \times 8^n}{(12)^{n+1}} = \frac{1}{3}$ « 2 »
5. $\frac{4^{2n} \times \left(\sqrt{3}\right)^{4n}}{9^n \times 4^n} = \frac{1}{16}$ « – 2 »
6. $\frac{4^{n-1} \times 2^{n+3}}{8^n} = 2n^2$ «±1»
7. 📖 $\frac{(14)^{2n} \times 4^{n+1}}{4 \times 7^n \times 16^n} = 49$ « 2 »

4 **Find the S.S. of each of the following equations in $\mathbb{R}$:**

1 $(x-4)^5 = 32$ « $\{6\}$ »

2 📖 $\frac{1}{(x+9)^4} = 0.0001$ « $\{1, -19\}$ »

3 $(x^2 - x)^5 = 32$ « $\{-1, 2\}$ »

4 $\left(\sqrt{3}\right)^{x^2 - x} = 1$ « $\{0, 1\}$ »

5 📖 $5^{x^2 - 5x} = 0.0016$ « $\{1, 4\}$ »

6 $5^{x^2} = 25^{x+4}$ « $\{4, -2\}$ »

7 📖 $9^{x^2 - 1} = \frac{1}{(27)^x}$ « $\{-2, \frac{1}{2}\}$ »

---

5 📖 If $\frac{6^{2n} \times 2^{2n}}{4^{2n} \times 3^{2n+4}} = 9^{-x}$, then find the value of : x « 2 »

---

6 If $\frac{(81)^x \times 4^x}{6^{2x} \times 3^{2x}} = 3^{y-1}$, find the value of : y « 1 »

---

7 If $\frac{(27)^{x-1} \times 8^x}{\left(2\sqrt{2}\right)^{2x} \times \left(3\sqrt{3}\right)^{2x}} = 3^x$, find the value of : x « – 3 »

---

8 If $\frac{7^x \times 6^x}{(14)^x} = 3^{2-y}$, find the value of : $x + y$ « 2 »

---

9 📖 If $\left(\sqrt{\frac{3}{2}}\right)^x = \frac{4}{9}$, calculate the value of : $\left(\frac{3}{2}\right)^{x+1}$ « $\frac{8}{27}$ »

---

10 📖 If $\frac{49^n \times 25^{2n} \times 3^{4n}}{7^{-n} \times 15^{4n}} = 343$, then calculate the value of : 6^{2n} « 36 »

---

11 📖 If $3^x = 27$, $4^{x+y} = 1$, calculate the value of each of : x and y « 3 , – 3 »

---

12 **Choose the correct answer from those given :**

1 If $3^{x+1} = 5^{x+1}$, then $x =$ ....

(a) 4 (b) 3 (c) – 1 (d) 1

2 If $3^{2+x} = 5^{x+2}$, then $7^{x+2} =$ ....

(a) 7 (b) – 7 (c) – 14 (d) 1

3 If $\left(\frac{2}{3}\right)^9 = \left(\frac{3}{2}\right)^x$, then $x =$ ....

(a) – 9 (b) 9 (c) 32 (d) 23



4 If $2^{x} = \frac{1}{8}$, then $x^{2} =$ ....

(a) $\frac{1}{4}$ (b) 9 (c) -9 (d) $-\frac{1}{9}$

5 If $5^{|x-3|} = 25$, then $x =$ ....

(a) 5 (b) 2 (c) 1 (d) 5 or 1

6 If $3^{x-1} = \sqrt[3]{\frac{1}{27}}$, then $x =$ ....

(a) 1 (b) zero (c) -1 (d) -2

7 If $\left(\sqrt{3}\right)^{x+1} = 3\sqrt{3}$, then $x =$ ....

(a) 1 (b) 2 (c) zero (d) 3

8 If $2^{x-2} = 2^{1-2x}$, then $x =$ ....

(a) 2 (b) $\frac{1}{2}$ (c) 1 (d) zero

9 If $3^{x} = 9$, then $2^{x} - 1 =$ ....

(a) 7 (b) 3 (c) 8 (d) 5

10 If $(x+1)^3 = \left(\frac{1}{8}\right)^{-3}$, then $x =$ ....

(a) -7 (b) 7 (c) zero (d) 9

11 If $2^{2x} = 4$, then $2^{5x} =$ ....

(a) 32 (b) 16 (c) 10 (d) 8

12 If $0.05 \times 0.002 = 10^{x}$, then $x =$ ....

(a) -4 (b) zero (c) 2 (d) 4

13 If $2^{x-1} \times 3^{1-x} = \frac{9}{4}$, then $x =$ ....

(a) -3 (b) -1 (c) 1 (d) 3

14 If $2^{x} = \left(2\sqrt{5} + 3\sqrt{2}\right)\left(2\sqrt{5} - 3\sqrt{2}\right)$, then $x =$ ....

(a) 1 (b) -1 (c) 2 (d) -2

15 If $3^{x} = 7$, $7^{y} = 9$, then $x\,y =$ ....

(a) 5 (b) 2 (c) 7 (d) 9

13 **Complete the following :**

1 If $5^{x(x-1)} = 1$, then the value of $x =$ ....

2 If $3^n \times 3^5 = 1$, then n = ...

3 If $2^y \times 5^y = 100$, then y = ...

4 If $\left(\frac{3}{5}\right)^{x-7} = 1$, then $x =$ ....

5 If $3^x \times 2^{-x} = 1.5$, then $x =$ ....

6 If $2^x \times 5^{-x} = 2.5$, then $x =$ ....

7 If $4^{x-6} = 64$, then $\sqrt{x} =$ ....

8 If $4^{x-10} = \frac{1}{16}$, then $\sqrt[3]{x} =$ ....

9 If $\left(\sqrt{5}\right)^{2x} = \frac{1}{5}$, then $x =$ ....

10 If $\frac{2^x \times 3^x}{12^x} = \frac{1}{2}$, then $x =$ ....

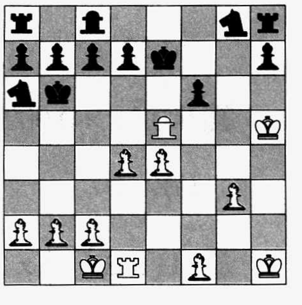
11 If $3^x + 3^x + 3^x = 1$, then $x =$ ....

12 If $\{3 , a^{x-2}\} = \{1 , 3\}$, then the value of $x =$ ....

13 If $(2^x , 125) = (16 , y^3)$, then $x =$ .... and y = ...

Life Application

14 If the area of Sameh's chess board is 2^n cm^2 , given that the side length of each small square which the board consists of is 4 cm., find the value of n « 10 »



For excellent pupils

15 **Find the value of x in each of the following where $x \in \mathbb{R}$:**

1 $x^{x+2} = 4^{x+2}$ « ± 4 or − 2 »

2 $a^{x+3} - 1 = (a-1)(a+1)(a^2+1)(a^4+1)$ « 5 »



From the school book

Exercise 12 On operations on integer powers

1 Complete the following :

1. The simplest form of the expression : $2^{-3} \times 2^{-2} \div 4^{-3} =$ ....
2. The simplest form of the expression : $2^{-3} \times 3^{-2} \div 6^{-4} =$ ....
3. The simplest form of the expression : $\left(3^{-2}\right)^3 \div 9^{-3} \times (-2)^{-1} =$ ....
4. The simplest form of the expression : $4^3 \times 3^{-2} \times \left(\sqrt[3]{-8}\right)^{-5} =$ ....

---

2 Find the result of each of the following in its simplest form :

1. $\left(\sqrt{5}\right)^5 \div 5\sqrt{5} + 2\sqrt{3} \times \sqrt{3}$ « 11 »
2. $\left(2\sqrt{3}\right)^3 \times \sqrt{3} - \left(\sqrt{2}\right)^7 \div 4\sqrt{2}$ « 70 »
3. $\left(\sqrt{3}\right)^{-3} \times 3\sqrt{3} + \left(\sqrt{3}\right)^{-4} \div \left(\sqrt{3}\right)^{-10}$ « 28 »
4. $\left(2\sqrt{5}\right)^4 - \left(\sqrt{5}\right)^{-3} \times \left(5\sqrt{5}\right)^2 \div 5\sqrt{5}$ « 399 »

---

3 Find the result of each of the following in its simplest form :

1. $\dfrac{\left(\sqrt{3}\right)^7 \times \left(\sqrt{3}\right)^{-5} - \left(\sqrt{3}\right)^2}{\left(\sqrt{3}\right)^7 \times \left(\sqrt{3}\right)^{-5} + \left(\sqrt{3}\right)^2}$ « zero »
2. $\dfrac{2\left(\sqrt{3}\right)^5 \div 3\sqrt{3}}{2\sqrt{3} + \left(\sqrt{3} - 1\right)^2}$ « $\frac{3}{2}$ »
3. $\dfrac{\left(2\sqrt{2}\right)^3 \times 3\sqrt{2}}{\left(\sqrt{6} + \sqrt{2}\right)^2 - 2\sqrt{12}}$ « 12 »

---

4 If $a = \sqrt{2}$, $b = \sqrt{3}$, find the numerical value of :

1. $\dfrac{b^4 - a^4}{b^2 + a^2}$ « 1 »
2. $\dfrac{a^3 + b^3}{a + b}$ « $5 - \sqrt{6}$ »

---

5 Choose the correct answer from those given :

1. The expression : $\dfrac{3^x \times 3^x \times 3^x}{3^x + 3^x + 3^x}$ equals ....

   (a) 3^{2x-1} (b) 3^{1-2x} (c) 3^{x^3-3x} (d) 3^{3x-x^3}

2. $(5^{x+2} - 5^{x+1}) \div 5^x =$ ....

   (a) 5 (b) 10 (c) 15 (d) 20

3 The value of the expression : $3^5 + (\sqrt{3})^{10} - 2(3)^5 = ...$

(a) zero (b) 3^5 (c) $(\sqrt{3})^5$ (d) $2(3)^5$

4 The simplest form of the expression : $\sqrt{4 \times \sqrt{16} \div \sqrt[3]{8} - 2^2} = ...$

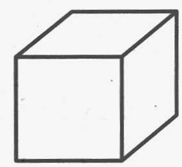
(a) 2 (b) 4 (c) 8 (d) 16

5 If $x = \sqrt{3}$, $y = \sqrt{5}$, then : $\frac{x^8 - y^8}{x^4 + y^4} = ...$

(a) 4 (b) – 4 (c) 16 (d) – 16

Geometric Applications

6 If the total area of a cube = 3.375×10^2 cm².



Find :

1 The edge length of the cube.

2 The volume of the cube.

« 7.5 cm. , 421.875 cm³. »

---

7 If the volume of the sphere = $\frac{4}{3} \pi r^3$



Find the radius length of the sphere whose volume is :

3.8808×10^4 cm³.

$\left(\pi = \frac{22}{7}\right)$

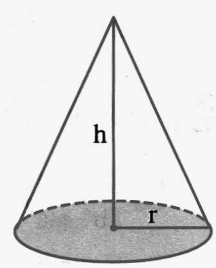
« 21 cm. »

---

8 If the volume of the right circular cone is given by the relation : $v = \frac{1}{3} \pi r^2 h$

Find the height of the cone h if the volume is : 7.7×10^2 cm³.

and its diameter length is 14 cm. $\left(\pi = \frac{22}{7}\right)$



« 15 cm. »

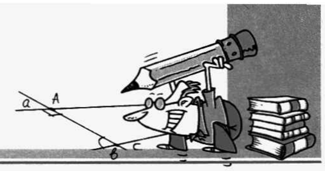
Life Applications

9 **Connecting with commercial business :**

If $c = m(1 + r)^n$ where (c) is the total sum (m) in pounds , (r) is the yearly profit per pound and (n) is the number of years , then calculate (c) to the nearest pound if :

$m = 2.5 \times 10^4$, $r = 9.8 \times 10^{-2}$, $n = 12$

« 76766 pounds »



10 **Population :**

If the number of population (y) in millions in a country is identified by the relation :

$y = 11.7\ (1.02)^{x}$ where x is the number of years starting from year 2005

Calculate the number of population expected for this country to the nearest million :

1 year 2011 **2** year 2000 « 13 millions , 11 millions »

For excellent pupils

11 If $x = 2 + \sqrt{3}$, $y = 2 - \sqrt{3}$

, then find the value of the expression $\dfrac{x^7 y^8 - y}{(x + y)^9}$ in the simplest form. « zero »

---

12 If $\ell = \frac{1}{2}\ (3^{x} + 3^{-x})$, $m = \frac{1}{2}\ (3^{x} - 3^{-x})$ **Prove that :** $\ell^2 - m^2 = 1$

General exarcise on unit two

from the school book

First : Completion questions

Complete the following :

1 The number $\left(\sqrt{2}\right)^{-3}$ in the simplest form is ....

2 The number $\frac{1}{\left(\sqrt{5}\right)^{-2}}$ in the simplest form is ....

3 $\left(\frac{2}{3}\right)^{-4} = \left(-\frac{\dots\dots\dots}{\dots\dots\dots}\right)^2$

4 If $3^{x-2} = 1$, then $x =$ ....

5 If $3^{x-1} = 27$, then $x =$ ....

6 The simplest form of the expression $\left(\sqrt{2}\right)^0 \times \left(\sqrt{2}\right) \times \left(\sqrt{2}\right)^2 \times \left(\sqrt{2}\right)^3$ is ....

7 The greater number $\left(-\sqrt{11}\right)^{24}$ or $\left(-\sqrt{11}\right)^{25}$ is ....

8 The simplest form of the expression $\left(\left(\sqrt{7}\right)^2\right)^3 - \left(\left(\sqrt{7}\right)^3\right)^2 =$ ....

9 The value of the expression $\sqrt[3]{\frac{216}{2^3 \times 3^3}} =$ ....

10 If five times a number is 5^3 , then $\frac{4}{5}$ of this number is ....

11 The simplest form of the expression : $2^0 + (2)^{-1} - \left(\frac{-1}{\sqrt{2}}\right)^2 =$ ....

12 If $x = \left(\sqrt{3} + 2\right)^9$, $y = \left(\sqrt{3} - 2\right)^9$, then xy equals ....

13 $x^{-2} + 1 = x^{-2}$ (.......... + ...) where $x \neq 0$

14 If $3^x \times 2^{-x} = 1.5$, then $x =$ ....

15 If $4^{x-10} = \frac{1}{16}$, then $\sqrt[3]{x} =$ ....

16 The simplest form of the expression : $2^{-3} \times (2)^{-2} \div 4^{-3} =$ ....

17 The simplest form of the expression : $(3^{-2})^3 \div 9^{-3} \times (-2)^{-1} =$ ....

18 The simplest form of the expression : $(2^3 \times 2^{-2})^7 \div \left(\sqrt[3]{-8}\right)^0 =$ ....

19 If $3^x + 3^x + 3^x = 1$, then $x =$ ....

20 If $\frac{2^x \times 3^x}{(12)^x} = \frac{1}{2}$, then $x =$ ....



Second : Choosing from multiple choices questions

Choose the correct answer from those given :

1 3^{-2} equals ....

(a) -9 (b) $-\frac{1}{9}$ (c) $\frac{1}{9}$ (d) 9

2 0.002×0.05 equals ....

(a) 10^{-5} (b) 10^{-4} (c) 10^{4} (d) 10^{5}

3 What is the nearest value of $11^2 + 9^2$?

(a) $22 + 18$ (b) $211 + 29$ (c) $120 + 80$ (d) $120 + 20$

4 The value of the expression : $2^{20} + 2^{21}$ equals ....

(a) 2×2^{40} (b) 2×2^{41} (c) 3×2^{20} (d) 3×2^{21}

5 One sixth of the number : $2^{12} \times 3^{12}$ is ....

(a) 6^2 (b) 6^4 (c) 6^{11} (d) 6^{23}

6 The value of the expression : $2^5 + \left(\sqrt{2}\right)^{10}$ equals ....

(a) 2^6 (b) 2^{10} (c) $\left(\sqrt{2}\right)^{15}$ (d) $\left(\sqrt{2}\right)^{20}$

7 $4^3 + 4^3 + 4^3 + 4^3$ equals ....

(a) 4^3 (b) 4^4 (c) 4^{12} (d) 4^{81}

8 $\left(\frac{\sqrt{5}}{3}\right)^{-2}$ equals ....

(a) $-\frac{9}{5}$ (b) $-\frac{5}{9}$ (c) $\frac{5}{9}$ (d) $\frac{9}{5}$

9 If $x = \frac{\sqrt{9}}{\sqrt{3}}$, then x^{-1} equals : ....

(a) $\frac{\sqrt{3}}{3}$ (b) $\frac{\sqrt{3}}{\sqrt{2}}$ (c) $\sqrt{3}$ (d) 2

10 If $6^x = 7$, then 6^{x+1} equals ....

(a) 8 (b) 13 (c) 36 (d) 42

11 If $3^x = 5$, then $(27)^x$ equals ....

(a) 9 (b) 25 (c) 125 (d) 729

12 If $5^x = 4$, then 5^{x-1} equals ....

(a) 1.25 (b) 0.8 (c) 0.125 (d) 0.08

13 If $9^{8-2x} = 1$, then x equals ....

(a) zero (b) $\frac{1}{4}$ (c) 4 (d) 6

14 If $(x-5)^0 = 1$, then $x \in$ ....

(a) $\mathbb{R} - \{5\}$ (b) $\mathbb{R} - \{-5\}$ (c) $\{5\}$ (d) $\mathbb{R}$

15 If $5^{x-3} = 1$, then $(2x)^2$ equals ....

(a) 36 (b) 9 (c) 4 (d) 3

16 If $3^x = 5$, $\frac{1}{3^y} = 7$, then $3^{x+y} =$ ....

(a) $\frac{5}{7}$ (b) $\frac{7}{5}$ (c) 2 (d) 12

17 If $2^{x-1} \times 3^{1-x} = \frac{9}{4}$, then $x =$ ....

(a) –3 (b) –1 (c) 1 (d) 3^{x-x^3}

18 The numerical value of the expression : $\frac{2^{2n+1} \times 5^{2n+1}}{10^{2n}}$ equals ....

(a) $\frac{1}{10}$ (b) 7 (c) 10 (d) 100

19 The expression : $(5^{x+2} - 5^{x+1}) \div 5^x =$ ....

(a) 5 (b) 10 (c) 15 (d) 20

20 The expression : $\frac{3^x \times 3^x \times 3^x}{3^x + 3^x + 3^x}$ equals ....

(a) 3^{2x-1} (b) 3^{1-2x} (c) 3^{x^3-3x} (d) 3^3

Third : Essay questions

1 Find the value of the following in the simplest form :

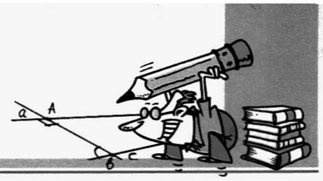
1 3^{-1}
2 $\left(\frac{1}{4}\right)^{-1}$
3 $\left(\frac{3}{2}\right)^{-3}$
4 $\left(\sqrt{5}\right)^4$
5 $\left(-\sqrt{3}\right)^{-2}$
6 $\left(\sqrt[3]{7}\right)^{-3}$
7 $\left(\frac{-1}{\sqrt{2}}\right)^6$
8 $(0.01)^{-2}$
9 $\left(-\frac{\sqrt{2}}{2}\right)^{-4}$

2 Find the value of each of the following in the simplest form :

1 $\left(\sqrt{3}\right)^{-2}$
2 $\left(-\sqrt[3]{4}\right)^{-3}$
3 $\left(\frac{1}{\sqrt{2}}\right)^{-3}$
4 $\left(\frac{\sqrt{3}}{3}\right)^{-5}$
5 $\left(\sqrt{3}\right)^{-4} \times \left(-\sqrt{2}\right)^4$
6 $\left(\frac{1}{\sqrt{3}}\right)^5 \div \left(\frac{1}{\sqrt{3}}\right)^7$

3 Simplify :

1 $\left(\sqrt{2}\right)^2 \times \left(\sqrt{2}\right)^4$
2 $\left(-\sqrt{5}\right)^9 \div \left(-\sqrt{5}\right)^5$
3 $\left(\sqrt{2}\right)^4 \times \left(\sqrt{3}\right)^4$
4 $\left(\left(\sqrt{3}\right)^2 \times \left(-\sqrt{2}\right)^3\right)^2$
5 $\left(\frac{2\sqrt{2}}{3\sqrt{3}}\right)^4$
6 $\frac{\left(\sqrt{3}\right)^7 \times \left(\sqrt{3}\right)^8}{\left(\sqrt{3}\right)^6}$



4 **Simplify each of the following in the simplest form :**

1. $\dfrac{(\sqrt{3})^{-5} \times (\sqrt{3})^{-4}}{(\sqrt{3})^{-10}}$
2. $\dfrac{(10)^2 \times (10)^{-7}}{(0.1)^2 \times 0.001}$
3. $\dfrac{(\sqrt{2})^{5} \times (3)^{-2}}{3 \times (\sqrt{2})^{9}}$
4. $\dfrac{(\sqrt{3})^{-3} \times (\sqrt{2})^{-4}}{(\sqrt{2} \times \sqrt{3})^{-5}}$

5 If : $x = 3$, $y = \sqrt{2}$, **find in the simplest form the value of each of the following :**

1. $x^{-2}\, y^{-4}$
2. $(x^{-2} \times y^{4})^{-2}$
3. $\left(\dfrac{x}{y}\right)^{-3}$

6 If : $x = \dfrac{\sqrt{3}}{2}$, $y = \dfrac{1}{\sqrt{3}}$, $z = \dfrac{\sqrt{2}}{2}$, **find the value of :** $x^2 + (x\,z)^2 \times y^2$

7 If : $x = 2$, $y = \sqrt{3}$ **find in the simplest form the value of :**

1. $(x + y)^4\,(x - y)^4$
2. $\left(\dfrac{x + y}{x - y}\right)^{-2}$

8 If : $a = \dfrac{1}{\sqrt{2}}$, $b = -1$, **find the value of :** $7\,a^6 + (1 - b)^{-3}$

9 If : $a = \sqrt{3}$, $b = \sqrt{2}$, **find the value of each of the following :**

1. $a^4 - b^4$
2. $\dfrac{a^4}{b^4}$

10 If : $x = 2\sqrt{2}$, $y = 3$, **find the value of :** $(x^2 - y^2)^3$

11 If : $\left(\sqrt{\dfrac{3}{2}}\right)^{x} = \dfrac{4}{9}$, **find the value of :** $\left(\dfrac{2}{3}\right)^{x+1}$

12 If : $x = \dfrac{\sqrt{3}}{2\sqrt{5}}$, $y = \dfrac{1}{\sqrt{2}}$, **Prove that :** $5\,x^2 + y^4 = 1$

13 If : $x = 2\sqrt{3}$, $y = \dfrac{4}{\sqrt{2}}$, **prove that :** $\sqrt{x^2 + y^4 + 5} = 9$

14 **Find the value of x in each of the following :**

1. $2^{x} = 32$
2. $2^{x-3} = 1$
3. $3^{x-2} = 81$
4. $(\sqrt{3})^{x-1} = 9$
5. $3^{x-2} = \dfrac{1}{9}$
6. $\left(\dfrac{2}{5}\right)^{2x-1} = \dfrac{8}{125}$

15 **Prove that :** $\dfrac{(27)^{x-1} \times 8^{x}}{\left(2\sqrt{2}\right)^{2x} \times \left(3\sqrt{3}\right)^{2x}} = \dfrac{1}{27}$

---

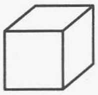
16 If : $\dfrac{8^{x} \times 9^{x}}{(18)^{x}} = 64$ **, find the value of :** $(4)^{-x}$

---

17 **Simplify :** $\dfrac{4^{x+1} \times 9^{2-x}}{6^{2x}}$, then calculate its value at : $x = 1$

---

18 If the total area of a cube equals : 3.375×10^{2} unit area **find :**



1. The length of the cube edge.
2. The volume of the cube.

---

19 If : $V = \dfrac{4}{3} \pi r^{3}$ is a rule between volume of a sphere V of radius r , find radius of a sphere of volume = 3.8808×10^{4} cm^{3}. $\left(\text{consider } \pi = \dfrac{22}{7}\right)$



Worksheet 5 till lesson (5) unit (1)

Answer the following questions :

1 Choose the correct answer from the given ones :

(1) $(x^3 + 64) \div (x + 4) = \cdots\cdots\cdots$

(a) $x^2 + 16$ (b) $x^2 - 4x + 16$

(c) $x^2 + 4x + 16$ (d) $x^2 - 4x - 16$

(2) If $x^3 + 8 = (x + 2)(x^2 + a + 4)$, then $a = \cdots\cdots\cdots$

(a) x (b) $-x$ (c) $-4x$ (d) $-2x$

(3) If $x^2 + e - 16 = (x + 4)(x - 4)$, then $e = \cdots\cdots\cdots$

(a) 8^x (b) zero (c) $-8x$ (d) $-4x$

(4) If $a - b = 5$, then $a^2 - 2ab + b^2 = \cdots\cdots\cdots$

(a) 25 (b) 20 (c) 15 (d) 10

(5) $3x^2y + 6xy = \cdots\cdots\cdots (x + 2)$

(a) $3x$ (b) $3xy^2$ (c) x^2y (d) $3xy$

(6) $(64)^2 - (36)^2 = \cdots\cdots\cdots$

(a) 100 (b) 28 (c) 2800 (d) 280

---

2 Complete the following :

(1) $2x^2 - 7x - 15 = (2x + 3)(\cdots\cdots\cdots)$

(2) If $kx^2 + 4x + 1$ is a perfect square , then $k = \cdots\cdots\cdots$

(3) If $x + y = 2$, $x^2 - xy + y^2 = 8$, then $x^3 + y^3 = \cdots\cdots\cdots$

(4) If $(a + b)^2 = 16$, $a^2 + b^2 = 8$, then $2ab = \cdots\cdots\cdots$

(5) If $(x + 2)$ is a factor of the expression : $x^2 - x - 6$, then the other factor is $\cdots\cdots\cdots$

(6) If $x^2 + ax + 5$ can be factorized , then $a = \cdots\cdots\cdots$

---

3 Factorize each of the following perfectly :

(1) $(x + 3)^2 - 25$ (2) $x^3 - 27$ (3) $2a^2 - 7ab + 6b^2$

---

4 [a] Use factorization to get the value of : $(99)^2 + 2 \times (99) + 1$

[b] Factorize the following expression perfectly : $(x^3 - 9)(x^3 + 9) + 17$



Worksheet 6 till lesson (6) unit (1)

Answer the following questions :

1 Choose the correct answer from the given ones :

(1) $(x - 2y)(x^2 + 2xy + 4y^2) = \cdots\cdots$

(a) $x^3 + 8y^3$ (b) $x^3 + 2y^3$ (c) $x^3 - 8y^3$ (d) $x^3 - 2y^3$

(2) If $a^2 + b^2 = 11$, $ab = 5$, then $a - b = \cdots\cdots$

(a) 6 (b) ± 1 (c) 1 (d) -1

(3) If $a + 2b = 4$, $x + y = 8$, then the numerical value of the expression $ax + ay + 2bx + 2by = \cdots\cdots$

(a) 2 (b) 12 (c) 4 (d) 32

(4) The expression : $x^2 - 6x + k$ is a perfect square when $k = \cdots\cdots$

(a) 36 (b) 9 (c) 3 (d) 12

(5) If $x + 3$ is a factor of the expression : $2x^2 + 11x + 15$, then the other factor is $\cdots\cdots$

(a) $x + 5$ (b) $2x + 5$ (c) $2x - 5$ (d) $x + 15$

(6) If $x^2 - kx + 5 = (x - 1)(x - 5)$, then $k = \cdots\cdots$

(a) 6 (b) 5 (c) 4 (d) -6

2 Complete the following :

(1) $x^2 - y^2 - x - y = (x + y)(\cdots\cdots)$

(2) If $x^2 + a = (x - 4)(x + 4)$, then $a = \cdots\cdots$

(3) $27x^3 + 8y^3 = (\cdots\cdots)(\cdots\cdots)$

(4) If $x - y = 2$, $x + y = 8$, then $x^2 - y^2 = \cdots\cdots$

(5) $3x^2 - 10x + 7 = (\cdots\cdots)(\cdots\cdots)$

(6) $x^2 + ax - bx - ab = \cdots\cdots$

3 Factorize each of the following perfectly :

(1) $\frac{1}{2}x^3 - 4$ (2) $4x^2 + 4xy + y^2 - 9$

(3) $x^3 - x - y + y^3$

4 [a] Use factorization to get the value of : $(8.175)^2 - (1.825)^2$

[b] Factorize the expression : $x^6 - y^6$ perfectly.



Worksheet 7 till lesson (7) unit (1)

Answer the following questions :

1 Choose the correct answer from the given ones :

(1) If $(x + y)^2 = 25$, $x y = 5$, then $x^2 + y^2 =$ ....

(a) 10 (b) 15 (c) 20 (d) 25

(2) The expression : $a x^2 - 40 x + 25$ is a perfect square when a = ...

(a) 2 (b) 9 (c) 4 (d) 16

(3) If $x + y = 3$, $x^2 - x y + y^2 = 5$, then $x^3 + y^3 =$ ....

(a) 15 (b) 25 (c) 8 (d) 27

(4) If $2 x^2 - c x - 2 = (2 x + 1)(x - 2)$, then c = ...

(a) 1 (b) 2 (c) 3 (d) 4

(5) The side length of the square whose area is $(x^2 + 6 x + 9)$ square units = ... length units. (where $x > -3$)

(a) $x + 3$ (b) $x + 9$ (c) $x^2 + 3$ (d) $x + 6$

(6) If $a - b = m$, $a + b = k$, then $a^2 - b^2 =$ ....

(a) m + k (b) m k (c) m − k (d) $\frac{m}{k}$

---

2 Complete the following :

(1) If $x^3 - k^3 = (x - 5)(x^2 + 5 x + 25)$, then k = ...

(2) $a x + a y + 2 x + 2 y = (x + y)$ (..........)

(3) The expression : $x^2 + 2 x + c$ can be factorized if c = ... , $c \in \mathbb{Z}_+$

(4) If $(x + 1)$ is a factor of the expression $(5 x^2 - 2 x - 7)$, then the other factor is ....

(5) If $(25)^2 - (15)^2 = 10 x$, then $x =$ ....

(6) $x^4 + 4 = (x^2 + 2)^2 -$ ....

---

3 Factorize each of the following perfectly :

(1) $3 x^3 - 75 x$

(2) $x^4 - x$

(3) $6 x^2 - 23 x - 18$

(4) $x y - 5 x - 6 y + 30$

---

4 Factorize each of the following by completing the square :

(1) $4 x^4 + 25 y^4 - 29 x^2 y^2$

(2) $x^2 (x^2 - 5 y^2) + 4 y^4$



Worksheet 8 till lesson (8) unit (1)

Answer the following questions :

1 Choose the correct answer from the given ones :

(1) The solution set of the equation : $x(x-3)=0$ in $\mathbb{R}$ is ………

(a) {3} (b) {0 , 3} (c) {0 , – 3} (d) {0}

(2) If $3x^2 + cx - 6 = (3x-2)(x+3)$, then c = ………

(a) 7 (b) 12 (c) 13 (d) 5

(3) The expression : $x^2 + 6x + a$ is a perfect square when a = ………

(a) 6 (b) 16 (c) 1 (d) 9

(4) $x^3 + y^3 = (\cdots\cdots)(x^2 - xy + y^2)$

(a) $x^2 + y^2$ (b) $x^2 - y^2$ (c) $x + y$ (d) $x - y$

(5) If the number 4 is a solution of the equation : $x^2 + x - 20 = 0$, then the other solution is ………

(a) 20 (b) 5 (c) – 5 (d) – 4

(6) One of the factors of the expression : $x^2 - 3x - 18$ is ………

(a) $x - 3$ (b) $x - 6$ (c) $x - 9$ (d) $x - 18$

---

2 Complete the following :

(1) $(\cdots\cdots - 3y)(2x + 5y) = 8x^2 + \cdots\cdots - 15y^2$

(2) If a + b = 5 , $x - y = 3$, then $ax - ay + bx - by = \cdots\cdots$

(3) If the number 9 is a solution of the equation : $x^2 + k = 0$, then k = ………

(4) The solution set of the equation : $x^2 + 25 = 0$ in $\mathbb{R}$ is ………

(5) If a – b = 3 , then $a^2 - 2ab + b^2 = \cdots\cdots$

(6) The solution set of the equation : $x^2 = 4x$ in $\mathbb{R}$ is ………

---

3 Factorize each of the following perfectly :

(1) $5x^2 - 4x - 12$

(2) $a^3 - 9a + a^2 - 9$

(3) $x^4 + 4y^4$

---

4 Find in $\mathbb{R}$ the solution set of each of the following two equations :

(1) $x^2 - 7x - 30 = 0$

(2) $x - \frac{3}{x} = 2$



Worksheet 9 till lesson (9) unit (1)

Answer the following questions :

1 Choose the correct answer from the given ones :

(1) The solution set of the equation : $x^2 - 5x = 0$ in $\mathbb{R}$ is ·········

(a) {5} (b) {0} (c) {0 , 5} (d) ∅

(2) $x^3 - 8 = (x - 2)$ (·········)

(a) $x^2 - 2x + 4$ (b) $x^2 - 4x + 4$ (c) $x^2 + 2x + 4$ (d) $x^2 + 4x + 4$

(3) If the expression : $x^2 + 7x + k$ can be factorized , then k = ·········

(a) 6 (b) 30 (c) – 12 (d) – 6

(4) If $x^2 + k x y - 25 y^2 = (x + 5y)(x - 5y)$, then k = ·········

(a) zero (b) 10 (c) – 10 (d) – 25

(5) If $y - x = 5$, $x^2 - y^2 = 10$, then $x + y$ = ·········

(a) 2 (b) – 2 (c) 5 (d) – 5

(6) If the age of Ayman 5 years ago was x years , then the square of his age now = ········· years.

(a) $x^2 + 5$ (b) $x^2 + 25$ (c) $(x + 5)^2$ (d) $(x - 5)^2$

---

2 Complete the following :

(1) If $(x + 2)$ is a factor of the expression : $2x^2 + 3x - 2$, then the other factor is ·········

(2) If $(x + y) = 3$, $(a + b) = 2$, then the value of the expression :
$a x + a y + b x + b y$ = ·········

(3) Twice the square of the number x is ·········

(4) If the expression : $a^2 x^2 + 10x + 25$ is a perfect square , then a = ·········

(5) If $x^3 + y^3 = 28$, $x + y = 2$, then $2x^2 - 2xy + 2y^2$ = ·········

(6) $x^2 - 5x - 6 = (x -$ ·········$)(x +$ ·········$)$

---

3 [a] Factorize each of the following expressions :

(1) $4x^2 - 4x - 15$ (2) $x^4 - 19x^2y^2 + 25y^4$

[b] Find in $\mathbb{R}$ the solution set of the equation : $x(3x - 7) = 6$

---

4 [a] Find the real number whose double exceeds its multiplicative inverse by one.

[b] A right-angled triangle , the lengths of the two sides of the right angle are $4x$ cm. and $x + 1$ cm. If the area of the triangle = 84 cm^2. , calculate the length of its hypotenuse.



Worksheet 10 till lesson (1) unit (2)

Answer the following questions :

1 Choose the correct answer from the given ones :

(1) $(\sqrt{3}+\sqrt{2})^9 (\sqrt{3}-\sqrt{2})^9 =$ ....

(a) 1 (b) $\sqrt{5}$ (c) $\sqrt{6}$ (d) 5

(2) If $x - y = 3$, $x^2 + xy + y^2 = 6$, then $x^3 - y^3 =$ ....

(a) 18 (b) 2 (c) 3 (d) 9

(3) $6x^2 - 11x + 3 =$ ....

(a) $(3x+1)(2x+3)$ (b) $(6x-1)(x-3)$

(c) $(3x-1)(2x-3)$ (d) $(x-6)(3x-1)$

(4) If $x - y = 2$, $x + y = 12$, then $x^2 - y^2 =$ ....

(a) 10 (b) 6 (c) 24 (d) 14

(5) $(999)^2 - 1 =$ ....

(a) 998 (b) $(1000)^2$ (c) $(998)^2$ (d) 998000

(6) If $2^x = 5$, then $2^{x+2} =$ ....

(a) 10 (b) 15 (c) 20 (d) 2

2 Complete the following :

(1) The S.S. of the equation : $3x^2 - 6x = 0$ in $\mathbb{R}$ is ....

(2) If $a^2 + ab + a + b = 10$, $a + b = 5$, then a = ...

(3) If $kx^2 + 4x + 1$ is a perfect square , then k = ...

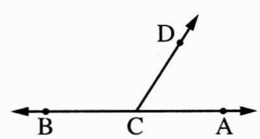
(4) If $(x - 5)$ is a factor of the expression : $x^2 - 4x - 5$, then the other factor is ....

(5) If : $(x-5)^0 = 1$, then $x \in$ ....

(6) **In the opposite figure :**

$\overleftrightarrow{AB} \cap \overrightarrow{CD} = \{C\}$, if m $(\angle ACD) = 8x$

, m $(\angle BCD) = x^2$, then $x =$ ....°



3 [a] Prove that : $\dfrac{(27)^{x-1} \times 8^x}{(2\sqrt{2})^{2x} \times (3\sqrt{3})^{2x}} = \dfrac{1}{27}$

[b] Factorize each of the following perfectly :

(1) $3ax + 2ay + 2by + 3bx$ (2) $8x^3 - 125$

4 [a] The length of a rectangle exceeds its width by 5 m. , if its area = 84 m^2 , find the two dimensions of the rectangle and its perimeter.

[b] If $x = 3$, $y = \sqrt{2}$, **then find in the simplest form :** $x^{-2} y^{-4}$



Worksheet 11 till lesson (2) unit (2)

Answer the following questions :

1 Choose the correct answer from the given ones :

(1) If $3^{x} = 27$, then $2^{x} - 3 = \cdots\cdots$

(a) 3 (b) 9 (c) 8 (d) 5

(2) $4^2 + 4^2 + 4^2 + 4^2 = \cdots\cdots$

(a) 4^5 (b) 16 (c) 4^8 (d) 4^3

(3) $(25)^2 - (15)^2 = 10 \times \cdots\cdots$

(a) 25 (b) 15 (c) 30 (d) 40

(4) If -1 is a root of the equation : $x^2 - 2x + m = 0$, then $m = \cdots\cdots$

(a) 1 (b) -1 (c) 3 (d) -3

(5) If $x^3 + 8 = (x + 2)(x^2 + a + 4)$, then $a = \cdots\cdots$

(a) $2x$ (b) $-2x$ (c) $4x$ (d) $-4x$

(6) The expression : $4x^2 + 2kx + 9$ is a perfect square if $k = \cdots\cdots$

(a) ± 2 (b) ± 3 (c) ± 6 (d) ± 9

---

2 Complete the following :

(1) $\left(\frac{2}{3}\right)^{-4} = \left(-\frac{\cdots\cdots}{\cdots\cdots}\right)^2$

(2) If $3^{x-2} = 1$, then $x = \cdots\cdots$

(3) If $x - 3y = 5$, $x + 3y = 7$, then $x^2 - 9y^2 = \cdots\cdots$

(4) If five times a number is 5^3 , then $\frac{4}{5}$ of this number is $\cdots\cdots$

(5) The S.S. of the equation : $x^2 - x = 0$ in $\mathbb{R}$ is $\cdots\cdots$

(6) If $4^{x-10} = \frac{1}{16}$, then $\sqrt[3]{x} = \cdots\cdots$

---

3 [a] Factorize each of the following perfectly :

(1) $2x^2 + x - 6$ (2) $8x^3 - 27$ (3) $x^2 - x + xy - y$

[b] Find the value of n if : $\frac{(14)^{2n} \times 4^{n+1}}{4 \times 7^n \times (16)^n} = 49$

---

4 [a] Factorize the expression : $x^4 + y^4 - 7x^2y^2$ by completing the square.

[b] If : $\left(\sqrt{\frac{3}{2}}\right)^{x} = \frac{4}{9}$ **, find the value of :** $\left(\frac{2}{3}\right)^{x+1}$



Worksheet 12 till lesson (3) unit (2)

Answer the following questions :

1 Choose the correct answer from those given :

(1) If $16x^2 + kx + 9$ is a perfect square, then k = ...

(a) ± 12 (b) ± 24 (c) ± 6 (d) ± 18

(2) If $x + y = 3$, $x^2 - xy + y^2 = 5$, then $x^3 + y^3 =$ ....

(a) 15 (b) 25 (c) 8 (d) 7

(3) If $6^x = 7$, then $6^{x+1} =$ ....

(a) 8 (b) 13 (c) 36 (d) 42

(4) If $3^x = 5$, $\frac{1}{3^y} = 7$, then $3^{x+y} =$ ....

(a) $\frac{5}{7}$ (b) $\frac{7}{5}$ (c) 2 (d) 12

(5) $\frac{3^x \times 3^x \times 3^x}{3^x + 3^x + 3^x} =$ ....

(a) 3^{2x-1} (b) 3^{1-2x} (c) 3^{x^3-3x} (d) 3^3

(6) The S.S. of the equation : $(x + 2)^2 - 25 = 0$ in $\mathbb{R}$ is ....

(a) {3, – 3} (b) {– 3, 7} (c) {3, 7} (d) {– 7, 3}

---

2 Complete the following :

(1) The simplest form of the expression : $2^{-3} \times 2^{-2} \div 4^{-3} =$ ....

(2) $x^2 - y^2 = (x - y)$ (..........)

(3) The S.S. of the equation : $x = \frac{1}{x}$ in $\mathbb{R}$ is ....

(4) If $(2x - 1)$ is a factor of the expression : $2x^2 + 5x - 3$, then the other factor is ....

(5) $a^2 - b^2 + a + b = (a + b)$ (..........)

(6) The side length of the square whose area is :
$(x^2 + 10x + 25)$ cm.^{2} = ... cm. (where $x > -5$)

---

3 Factorize each of the following perfectly :

(1) $2x^2 - 50y^2$

(2) $5x^2 - 8xy - 21y^2$

(3) $8x^3 - 343x^6$

(4) $x^4 + 4\ell^4$

---

4 [a] If : $x = \frac{\sqrt{3}}{2}$, $y = \frac{1}{\sqrt{3}}$ and $z = \frac{\sqrt{2}}{2}$, **find the value of :** $x^2 + (xz)^2 \times y^2$

[b] If $\frac{8^x \times 9^x}{(18)^x} = 64$, **find the value of :** 4^{-x}

4 $\left(3a+(2a+b)\right)\left(3a-(2a+b)\right)$
$=(5a+b)(a-b)$

5 $\left(ab+(ab-1)\right)\left(ab-(ab-1)\right)=(2ab-1)$

6 $\left((x+1)+(x-1)\right)\left((x+1)-(x-1)\right)$
$=2x\times 2=4x$

7 $\left(3(m-1)+5(m+1)\right)\left(3(m-1)-5(m+1)\right)$
$=(3m-3+5n+5)(3m-3-5m-5)$
$=(8m+2)(-2m-8)=-4(4m+1)(m+4)$

8 $\left((x+y+5)+(x-y-5)\right)\left((x+y+5)-(x-y-5)\right)$
$=2x(2y+10)=4x(y+5)$

9 $a^2-4b^2-5b^2=a^2-9b^2$
$=(a-3b)(a+3b)$

5

1 $(77+23)(77-23)=100\times 54=5400$

2 $(75+25)(75-25)=100\times 50=5000$

3 $(78+77)(78-77)=155\times 1=155$

4 $(125+25)(125-25)=150\times 100=15000$

5 $(11.6+1.6)(11.6-1.6)=13.2\times 10=132$

6 $(8.27+1.73)(8.27-1.73)=10\times 6.54=65.4$

7 $(95+5)(95-5)=100\times 90=9000$

8 $(999+1)(999-1)=1000\times 998=998000$

9 $2\left[(25.87)^2-(24.13)^2\right]$
$=2(25.87+24.13)(25.87-24.13)$
$=2\times 50\times 1.74=174$

6

1 $(30+1)(30-1)=900-1=899$

2 $(100+3)(100-3)=(100)^2-(3)^2=10000-9$
$=9991$

7

The expression $=\left((x+y)+(x-y)\right)\left((x+y)-(x-y)\right)$
$=2x\times 2y=4xy=4\times 8=32$

8

1 $3y, 2x, 9y^2$ 2 $5x, 5x, 9m^2$
3 $16.8x, 8x$ 4 6 5 2
6 3 7 1 8 9
9 28

9

1 c 2 a 3 c 4 c 5 d
6 c 7 a 8 a 9 b

10

Let the length of the other side $=x$

$\therefore x^2=(41)^2-(40)^2=(41-40)(41+40)=1\times 81=81$

$\therefore x=\sqrt{81}=9$ cm.

$\therefore$ the length of the other side = 9 cm.

11

1 $b(a+2b)\left((a+2b)^2-b^2\right)$
$=b(a+2b)\left((a+2b)+b\right)\left((a+2b)-b\right)$
$=b(a+2b)(a+3b)(a+b)$

2 $(a-b)^2-c^2=\left((a-b)+c\right)\left((a-b)-c\right)$
$=(a-b+c)(a-b-c)$

3 $(2a+3b)^3-4a^2(2a+3b)$
$=(2a+3b)\left((2a+3b)^2-4a^2\right)$
$=(2a+3b)\left((2a+3b)-2a\right)\left((2a+3b)+2a\right)$
$=3b(2a+3b)(4a+3b)$

12

$$x=\frac{(225)^2-(25)^2}{25\times 20}=\frac{(225+25)(225-25)}{25\times 20}$$
$$=\frac{250\times 200}{25\times 20}=100$$

13

$(x-y)^2=4$ taking the square root of the two sides

$x-y=2$ where $x>y$

$\therefore (x^2-y^2)=(x+y)(x-y)=8\times 2=16$

Answers of Exercise 5

1

1 $(x+2)(x^2-2x+4)$

2 $(x-1)(x^2+x+1)$

3 $(4x+3)(16x^2-12x+9)$

4 $(2x-5)(4x^2+10x+25)$

5 $(5+a)(25-5a+a^2)$

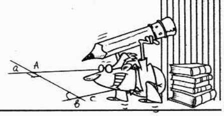
6 $(7-3m)(49+21m+9m^2)$

7 $(m+4n)(m^2-4mn+16n^2)$

8 $(8x-y)(64x^2+8xy+y^2)$

9 $(xy+3)(x^2y^2-3xy+9)$

10 $(3xy-4)(9x^2y^2+12xy+16)$



11 $\left(\frac{1}{2}a - 2b\right)\left(\frac{1}{4}a^2 + ab + 4b^2\right)$

12 $\left(\ell - \frac{1}{5}\right)\left(\ell^2 + \frac{1}{5}\ell + \frac{1}{25}\right)$

13 $(2a + 0.1)(4a^2 - 0.2a + 0.01)$

14 $(0.3m - n)(0.09m^2 + 0.3mn + n^2)$

15 $(1 + 5b^2)(1 - 5b^2 + 25b^4)$

16 $(2x - 7y^2)(4x^2 + 14xy^2 + 49y^4)$

17 $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$

18 $(x^3 - 8)(x^3 + 8)$

$= (x - 2)(x^2 + 2x + 4)(x + 2)(x^2 - 2x + 4)$

2

1 $2(x^3 + 8) = 2(x + 2)(x^2 - 2x + 4)$

2 $3(x^3 - 27) = 3(x - 3)(x^2 + 3x + 9)$

3 $\ell(\ell^3 + 64) = \ell(\ell + 4)(\ell^2 - 4\ell + 16)$

4 $m(\ell^3 - 27m^3) = m(\ell - 3m)(\ell^2 + 3\ell m + 9m^2)$

5 $3x(x^3 + 1) = 3x(x + 1)(x^2 - x + 1)$

6 $2x^2(x^3 - 27) = 2x^2(x - 3)(x^2 + 3x + 9)$

7 $2(8x^3 + 125y^3)$

$= 2(2x + 5y)(4x^2 - 10xy + 25y^2)$

8 $2b(8a^3 + 343b^3)$

$= 2b(2a + 7b)(4a^2 - 14ab + 49b^2)$

9 $2xy^2(27x^3 - 8y^3)$

$= 2xy^2(3x - 2y)(9x^2 + 6xy + 4y^2)$

10 $4x^5y^2(125x^3 - 64y^3)$

$= 4x^5y^2(5x - 4y)(25x^2 + 20xy + 16y^2)$

11 $\frac{1}{2}(x^3 + 8) = \frac{1}{2}(x + 2)(x^2 - 2x + 4)$

12 $\frac{1}{3}(x^3 - 27) = \frac{1}{3}(x - 3)(x^2 + 3x + 9)$

3

1 a	**2** a	**3** b	**4** c
5 d	**6** b	**7** b	**8** c

4

1 $x^2 + x + 1$

2 $(\mathbf{2a} + \mathbf{5})(4a^2 - 10a + \mathbf{25})$

3 $(\mathbf{x^4} + \mathbf{y^5})(\mathbf{x^8} - \mathbf{x^4y^5} + \mathbf{y^{10}})$

4 $8a^3 - \mathbf{27} = (\mathbf{2a} - \mathbf{3})(\mathbf{4a^2} + \mathbf{6a} + 9)$

5 $x^2 + 3x + 9$

6 $2a + 1$

5

$x^2 - y^2 = (x - y)(x + y)$

$20 = 2(x + y)$ $\quad \because x + y = 10$

$\therefore x^3 + y^3 = (x + y)(x^2 - xy + y^2) = 10 \times 28 = 280$

6

1 $\left((a + b) - b\right)\left((a + b)^2 + b(a + b) + b^2\right)$

$= a(a^2 + 2ab + b^2 + ba + b^2 + b^2)$

$= a(a^2 + 3ab + 3b^2)$

2 $\left((x + 5) - 5\right)\left((x + 5)^2 + 5(x + 5) + 25\right)$

$= x(x^2 + 10x + 25 + 5x + 25 + 25)$

$= x(x^2 + 15x + 75)$

3 $\left((m - 2n) - 2n\right) \times \left((m - 2n)^2 + 2n(m - 2n) + 4n^2\right)$

$= (m - 4n)(m^2 - 4mn + 4n^2 + 2mn - 4n^2 + 4n^2)$

$= (m - 4n)(m^2 - 2mn + 4n^2)$

4 $\left(2(m + n) - n\right)\left(4(m + n)^2 + 2n(m + n) + n^2\right)$

$= (2m + 2n - n)$

$(4m^2 + 8mn + 4n^2 + 2mn + 2n^2 + n^2)$

$= (2m + n)(4m^2 + 10mn + 7n^2)$

5 $2\left(1 - (x - 1)^3\right) = 2\left(1 - (x - 1)\right)\left(1 + (x - 1) + (x - 1)^2\right)$

$= 2(2 - x)(1 + x - 1 + x^2 - 2x + 1)$

$= 2(2 - x)(x^2 - x + 1)$

6 $\left((x + 5) + (x - 5)\right)$

$\left((x + 5)^2 - (x + 5)(x - 5) + (x - 5)^2\right)$

$= 2x(x^2 + 10x + 25 - x^2 + 25 + x^2 - 10x + 25)$

$= 2x(x^2 + 75)$

7 $\left((x + y) - (x - y)\right)$

$\left((x + y)^2 + (x + y)(x - y) + (x - y)^2\right)$

$= 2y(x^2 + 2xy + y^2 + x^2 - y^2 + x^2 - 2xy + y^2)$

$= 2y(3x^2 + y^2)$

8 $(m - n)\left(1 + (m - n)^3\right)$

$= (m - n)(1 + (m - n))\left(1 - (m - n) + (m - n)^2\right)$

$= (m - n)(m - n + 1)\left(1 - m + n + m^2 - 2mn + n^2\right)$

9 $(x^6 - 4) - 4 = x^6 - 8 = (x^2 - 2)(x^4 + 2x^2 + 4)$

10 $x^3 - 27 + 28 = x^3 + 1 = (x + 1)(x^2 - x + 1)$

7

1 $(m^3 - 1)(m^3 - 2) = (m - 1)(m^2 + m + 1)(m^3 - 2)$

2 $(y^3-1)(y^3+27)$
$=(y-1)(y^2+y+1)(y+3)(y^2-3y+9)$

3 $(x^3-1)(x^3-27)$
$=(x-1)(x^2+x+1)(x-3)(x^2+3x+9)$

4 $(x^3+1)(x^3-8)$
$=(x+1)(x^2-x+1)(x-2)(x^2+2x+4)$

8

$(x+5)^4-(x+5)=(x+5)\left((x+5)^3-1\right)$
$=(x+5)\left((x+5)-1\right)\left((x+5)^2+(x+5)+1\right)$
$=(x+5)(x+4)(x^2+10x+25+x+5+1)$
$=(x+5)(x+4)(x^2+11x+31)$

9

$x-y=1$
$\therefore$ squaring the two sides $(x-y)^2=1$
$\therefore x^2-2xy+y^2=1$ $\quad\therefore x^2+y^2=1+2xy$
$\because xy=2$ $\quad\therefore x^2+y^2=1+4=5$
$\therefore (x^3-y^3)=(x-y)(x^2+xy+y^2)$
$=1\times(5+2)=1\times7=7$

10

Let the two numbers be x and y
$\because x+y=2$ $\quad\therefore (x+y)^2=4$
$\because x^2+2xy+y^2=4$, $\because x^2+y^2=34$
$\therefore 34+2xy=4$ $\quad\therefore xy=-15$
$\therefore x^3+y^3=(x+y)(x^2-xy+y^2)$
$=2(34+15)=98$

Answers of Exercise 6

1

1. $x(a+b)+y(a+b)=(a+b)(x+y)$
2. $b(a-d)+h(a-d)=(a-d)(b+h)$
3. $x(a+y)+(a+y)=(a+y)(x+1)$
4. $a(m-n)+(m-n)=(m-n)(a+1)$
5. $x(a-c)+y(a-c)=(a-c)(x+y)$
6. $m(x-y)-n(x-y)=(x-y)(m-n)$
7. $y(x+5)+7(x+5)=(x+5)(y+7)$
8. $7(x-4)+a(x-4)=(x-4)(7+a)$
9. $5(\ell-2m)-a(\ell-2m)=(\ell-2m)(5-a)$
10. $a(3x-1)-2b(3x-1)=(3x-1)(a-2b)$

2

1. $c(c+d)+h(c+d)=(c+d)(c+h)$
2. $2m(3m+1)-n(3m+1)=(3m+1)(2m-n)$
3. $2m(4n-m)+3\ell(4n-m)=(4n-m)(2m+3\ell)$
4. $x(x-2z)-2y(x-2z)=(x-2z)(x-2y)$
5. $(a+b)^2-c^2=(a+b-c)(a+b+c)$
6. $(5x-1)^2-y^2=(5x-1-y)(5x-1+y)$
7. $1-(x+2y)^2=(1-x-2y)(1+x+2y)$
8. $(x-y)(x+y)+4(x+y)=(x+y)(x-y+4)$
9. $(x-2y)(x+2y)-5(x-2y)$
   $=(x-2y)(x+2y-5)$
10. $(3x+y)^2-4a^2=(3x+y-2a)(3x+y+2a)$
11. $2x(xy+a)-y(xy+a)=(xy+a)(2x-y)$
12. $bx(ax+1)-(ax+1)=(ax+1)(bx-1)$

3

1. $a^2(a+1)+(a+1)=(a+1)(a^2+1)$
2. $x^2(x-3)+6(x-3)=(x-3)(x^2+6)$
3. $(a+b)(a^2-ab+b^2)-(a+b)$
   $=(a+b)(a^2-ab+b^2-1)$
4. $x^2(x+2)-(x+2)=(x+2)(x^2-1)$
   $=(x+2)(x-1)(x+1)$
5. $a^2(a+1)-9(a+1)=(a+1)(a^2-9)$
   $=(a+1)(a-3)(a+3)$
6. $x^2(3x+2)+4(3x+2)=(3x+2)(x^2+4)$
7. $(y^3+8)+6y(y+2)$
   $=(y+2)(y^2-2y+4)+6y(y+2)$
   $=(y+2)(y^2-2y+4+6y)$
   $=(y+2)(y^2+4y+4)$
   $=(y+2)(y+2)^2=(y+2)^3$
8. $a(a^3-3a^2-15+5a)$
   $=a\left(a^2(a-3)+5(a-3)\right)$
   $=a(a-3)(a^2+5)$
9. $a^2(a^3-2)+(a^3-2)=(a^3-2)(a^2+1)$
10. $x^2(y^3+8)-(y^3+8)=(y^3+8)(x^2-1)$
   $=(y+2)(y^2-2y+4)(x-1)(x+1)$

4

1. $x^3(x^2-1)-(x^2-1)=(x^2-1)(x^3-1)$
   $=(x-1)(x+1)(x-1)(x^2+x+1)$
   $=(x-1)^2(x+1)(x^2+x+1)$

2 $4m^4 - (9m^2 - 6m + 1) = 4m^4 - (3m - 1)^2$

$= \left(2m^2 - (3m - 1)\right)\left(2m^2 + (3m - 1)\right)$

$= (2m^2 - 3m + 1)(2m^2 + 3m - 1)$

$= (2m - 1)(m - 1)(2m^2 + 3m - 1)$

3 $121x^4 - (100x^2 + 20x + 1)$

$= 121x^4 - (10x + 1)^2$

$= (11x^2 - 10x - 1)(11x^2 + 10x + 1)$

$= (11x + 1)(x - 1)(11x^2 + 10x + 1)$

5

$\because x^3 + 5x^2 + 3x + 15 = (x^3 + 5x^2) + (3x + 15)$

$= x^2(x + 5) + 3(x + 5)$

$= (x + 5)(x^2 + 3)$

, $\because$ $(x + 5)$ cm. is the width of the rectangle

$\therefore$ length of the rectangle $= (x^2 + 3)$ cm.

$\therefore$ perimeter of the rectangle $= 2(x + 5 + x^2 + 3)$

$= 2(x^2 + x + 8)$ cm.

, when $x = 2$

$\therefore$ perimeter of the rectangle $= 2(4 + 2 + 8) = 28$ cm.

6

$BC = \dfrac{x(x^2 + 2) + 2(x^2 + 2)}{x + 2}$

$= \dfrac{(x^2 + 2)(x + 2)}{x + 2} = (x^2 + 2)$ cm.

7

1 $2x\left(x^2(x + 3) - 9x - 27\right)$

$= 2x\left(x^2(x + 3) - 9(x + 3)\right)$

$= 2x(x + 3)(x^2 - 9) = 2x(x + 3)(x - 3)(x + 3)$

$= 2x(x - 3)(x + 3)^2$

2 $a^2 + 4ab + 4b^2 - 9 = (a + 2b)^2 - 9$

$= (a + 2b - 3)(a + 2b + 3)$

3 $a^3 - 4b^3 + a^2b - 4ab^2 = a^2(a + b) - 4b^2(a + b)$

$= (a + b)(a^2 - 4b^2) = (a + b)(a - 2b)(a + 2b)$

4 $a^2(b - 5) - 7a(b - 5) - 18(b - 5)$

$= (b - 5)(a^2 - 7a - 18) = (b - 5)(a - 9)(a + 2)$

8

1 $(x - 2y)^2 + (x - 2y) = (x - 2y)(x - 2y + 1)$

2 $3(x^2 - 5x - 24) - y(x - 8)$

$= 3(x - 8)(x + 3) - y(x - 8)$

$= (x - 8)\left(3(x + 3) - y\right) = (x - 8)(3x + 9 - y)$

3 $a^3 - 1 + a - 1 = (a - 1)(a^2 + a + 1) + (a - 1)$

$= (a - 1)(a^2 + a + 1 + 1) = (a - 1)(a^2 + a + 2)$

4 $a^3 + 8 + a^2 - 4$

$= (a + 2)(a^2 - 2a + 4) + (a - 2)(a + 2)$

$= (a + 2)(a^2 - 2a + 4 + a - 2) = (a + 2)(a^2 - a + 2)$

Answers of Exercise 7

1

1 $x^4 + 4 + 4x^2 - 4x^2 = (x^4 + 4x^2 + 4) - 4x^2$

$= (x^2 + 2)^2 - 4x^2 = (x^2 + 2 - 2x)(x^2 + 2 + 2x)$

$= (x^2 - 2x + 2)(x^2 + 2x + 2)$

2 $x^4 + 16x^2 + 64 - 16x^2$

$= (x^2 + 8)^2 - 16x^2 = (x^2 + 8 - 4x)(x^2 + 8 + 4x)$

$= (x^2 - 4x + 8)(x^2 + 4x + 8)$

3 $x^4 + 4x^2y^2 + 4y^4 - 4x^2y^2 = (x^2 + 2y^2)^2 - 4x^2y^2$

$= (x^2 + 2y^2 - 2xy)(x^2 + 2y^2 + 2xy)$

$= (x^2 - 2xy + 2y^2)(x^2 + 2xy + 2y^2)$

4 $x^4 + 16x^2y^2 + 64y^4 - 16x^2y^2$

$= (x^2 + 8y^2)^2 - 16x^2y^2$

$= (x^2 + 8y^2 - 4xy)(x^2 + 8y^2 + 4xy)$

$= (x^2 - 4xy + 8y^2)(x^2 + 4xy + 8y^2)$

5 $a^4 + 100a^2b^2 + 2500b^4 - 100a^2b^2$

$= (a^2 + 50b^2)^2 - 100a^2b^2$

$= (a^2 + 50b^2 - 10ab)(a^2 + 50b^2 + 10ab)$

$= (a^2 - 10ab + 50b^2)(a^2 + 10ab + 50b^2)$

6 $81x^4 + 36x^2z^2 + 4z^4 - 36x^2z^2$

$= (9x^2 + 2z^2)^2 - 36x^2z^2$

$= (9x^2 + 2z^2 - 6xz)(9x^2 + 2z^2 + 6xz)$

$= (9x^2 - 6xz + 2z^2)(9x^2 + 6xz + 2z^2)$

7 $4x^4 + 100x^2z^2 + 625z^4 - 100x^2z^2$

$= (2x^2 + 25z^2)^2 - 100x^2z^2$

$= (2x^2 + 25z^2 - 10xz)(2x^2 + 25z^2 + 10xz)$

$= (2x^2 - 10xz + 25z^2)(2x^2 + 10xz + 25z^2)$

8 $64x^4 + 144x^2y^2 + 81y^4 - 144x^2y^2$

$= (8x^2 + 9y^2)^2 - 144x^2y^2$

$= (8x^2 + 9y^2 - 12xy)(8x^2 + 9y^2 + 12xy)$

$= (8x^2 - 12xy + 9y^2)(8x^2 + 12xy + 9y^2)$

9 $3(4x^4+y^4)=3(4x^4+4x^2y^2+y^4-4x^2y^2)$
$=3\left((2x^2+y^2)^2-4x^2y^2\right)$
$=3(2x^2+y^2-2xy)(2x^2+y^2+2xy)$
$=3(2x^2-2xy+y^2)(2x^2+2xy+y^2)$

10 $2y^2(4x^4+81z^4)$
$=2y^2(4x^4+36x^2z^2+81z^4-36x^2z^2)$
$=2y^2\left((2x^2+9z^2)^2-36x^2z^2\right)$
$=2y^2(2x^2+9z^2-6xz)(2x^2+9z^2+6xz)$
$=2y^2(2x^2-6xz+9z^2)(2x^2+6xy+9z^2)$

2

1 $9x^4+6x^2+1+2x^2-6x^2$
$=(3x^2+1)^2-4x^2=(3x^2+1-2x)(3x^2+1+2x)$
$=(3x^2-2x+1)(3x^2+2x+1)$

2 $x^4+8x^2+16-28x^2-8x^2$
$=(x^2+4)^2-36x^2=(x^2+4-6x)(x^2+4+6x)$
$=(x^2-6x+4)(x^2+6x+4)$

3 $x^4+18x^2+81+9x^2-18x^2$
$=(x^2+9)^2-9x^2=(x^2+9-3x)(x^2+9+3x)$
$=(x^2-3x+9)(x^2+3x+9)$

4 $9x^4-24x^2+16-25x^2+24x^2$
$=(3x^2-4)^2-x^2=(3x^2-4-x)(3x^2-4+x)$
$=(3x^2-x-4)(3x^2+x-4)$
$=(3x-4)(x+1)(3x+4)(x-1)$

5 $x^4+4x^2y^2+4y^4+3x^2y^2-4x^2y^2$
$=(x^2+2y^2)^2-x^2y^2$
$=(x^2+2y^2-xy)(x^2+2y^2+xy)$
$=(x^2-xy+2y^2)(x^2+xy+2y^2)$

6 $m^4-2m^2n^2+n^4-11m^2n^2+2m^2n^2$
$=(m^2-n^2)^2-9m^2n^2$
$=(m^2-n^2-3mn)(m^2-n^2+3mn)$
$=(m^2-3mn-n^2)(m^2+3mn-n^2)$

7 $x^4+10x^2y^2+25y^4+x^2y^2-10x^2y^2$
$=(x^2+5y^2)^2-9x^2y^2$
$=(x^2+5y^2-3xy)(x^2+5y^2+3xy)$
$=(x^2-3xy+5y^2)(x^2+3xy+5y^2)$

8 $a^4+8a^2b^2+16b^4+4a^2b^2-8a^2b^2$
$=(a^2+4b^2)^2-4a^2b^2$
$=(a^2+4b^2-2ab)(a^2+4b^2+2ab)$
$=(a^2-2ab+4b^2)(a^2+2ab+4b^2)$

9 $x^4+2x^2y^2+y^4-7x^2y^2-2x^2y^2$
$=(x^2+y^2)^2-9x^2y^2$
$=(x^2+y^2-3xy)(x^2+y^2+3xy)$
$=(x^2-3xy+y^2)(x^2+3xy+y^2)$

10 $16x^4-24x^2y^2+9y^4-28x^2y^2+24x^2y^2$
$=(4x^2-3y^2)^2-4x^2y^2$
$=(4x^2-3y^2-2xy)(4x^2-3y^2+2xy)$
$=(4x^2-2xy-3y^2)(4x^2+2xy-3y^2)$

11 $4x^4-20x^2y^2+25y^4-29x^2y^2+20x^2y^2$
$=(2x^2-5y^2)^2-9x^2y^2$
$=(2x^2-5y^2-3xy)(2x^2-5y^2+3xy)$
$=(2x^2-3xy-5y^2)(2x^2+3xy-5y^2)$
$=(2x-5y)(x+y)(2x+5y)(x-y)$

12 $3(m^4-2m^2n^2+n^4-16m^2n^2)$
$=3\left((m^2-n^2)^2-16m^2n^2\right)$
$=3(m^2-n^2-4mn)(m^2-n^2+4mn)$
$=3(m^2-4mn-n^2)(m^2+4mn-n^2)$

13 $2(25x^4+9y^4-34x^2y^2)$
$=2(25x^4-30x^2y^2+9y^4-34x^2y^2+30x^2y^2)$
$=2\left((5x^2-3y^2)^2-4x^2y^2\right)$
$=2(5x^2-3y^2-2xy)(5x^2-3y^2+2xy)$
$=2(5x^2-2xy-3y^2)(5x^2+2xy-3y^2)$
$=2(5x+3y)(x-y)(5x-3y)(x+y)$

14 $2a(9b^4-57b^2c^2+64c^4)$
$=2a(9b^4-48b^2c^2+64c^4-57b^2c^2+48b^2c^2)$
$=2a\left((3b^2-8c^2)^2-9b^2c^2\right)$
$=2a(3b^2-8c^2-3bc)(3b^2-8c^2+3bc)$
$=2a(3b^2-3bc-8c^2)(3b^2+3bc-8c^2)$

3

1 $9x^4-10x^2y^2+y^4$
$=9x^4-6x^2y^2+y^4-10x^2y^2+6x^2y^2$
$=(3x^2-y^2)^2-4x^2y^2$
$=(3x^2-y^2-2xy)(3x^2-y^2+2xy)$
$=(3x^2-2xy-y^2)(3x^2+2xy-y^2)$
$=(3x+y)(x-y)(3x-y)(x+y)$

2 $x^4-19x^2y^2+25y^4$
$=x^4-10x^2y^2+25y^4-19x^2y^2+10x^2y^2$
$=(x^2-5y^2)^2-9x^2y^2$
$=(x^2-5y^2-3xy)(x^2-5y^2+3xy)$
$=(x^2-3xy-5y^2)(x^2+3xy-5y^2)$

3 $16x^4 - 28x^2y^2 + y^4$

$= 16x^4 + 8x^2y^2 + y^4 - 28x^2y^2 - 8x^2y^2$

$= (4x^2 + y^2)^2 - 36x^2y^2$

$= (4x^2 + y^2 - 6xy)(4x^2 + y^2 + 6xy)$

$= (4x^2 - 6xy + y^2)(4x^2 + 6xy + y^2)$

4 $4a^4 - 24a^2b^2 + 9b^4$

$= 4a^4 + 12a^2b^2 + 9b^4 - 24a^2b^2 - 12a^2b^2$

$= (2a^2 + 3b^2)^2 - 36a^2b^2$

$= \left(2a^2 + 3b^2 + 6ab\right)\left(2a^2 + 3b^2 + 6ab\right)$

$= (2a^2 - 6ab + 3b^2)(2a^2 + 6ab + 3b^2)$

4

1 $(x^4 - 4y^4)(x^4 + 4y^4)$

$= (x^2 - 2y^2)(x^2 + 2y^2)(x^4 + 4y^4)$

$= (x^2 - 2y^2)(x^2 + 2y^2)(x^4 + 4x^2y^2 + 4y^4 - 4x^2y^2)$

$= (x^2 - 2y^2)(x^2 + 2y^2)\left((x^2 + 2y^2)^2 - 4x^2y^2\right)$

$= (x^2 - 2y^2)(x^2 + 2y^2)(x^2 + 2y^2 - 2xy)$
$(x^2 + 2y^2 + 2xy)$

$= (x^2 - 2y^2)(x^2 + 2y^2)(x^2 - 2xy + 2y^2)$
$(x^2 + 2xy + 2y^2)$

2 $(x^4 - 25)(x^4 + 4)$

$= (x^2 - 5)(x^2 + 5)(x^4 + 4x^2 + 4 - 4x^2)$

$= (x^2 - 5)(x^2 + 5)\left((x^2 + 2)^2 - 4x^2\right)$

$= (x^2 - 5)(x^2 + 5)(x^2 + 2 - 2x)(x^2 + 2 + 2x)$

$= (x^2 - 5)(x^2 + 5)(x^2 - 2x + 2)(x^2 + 2x + 2)$

3 $(x^4 - 9y^4)(x^4 + 4y^4)$

$= (x^2 - 3y^2)(x^2 + 3y^2)$
$\times (x^4 + 4x^2y^2 + 4y^4 - 4x^2y^2)$

$= (x^2 - 3y^2)(x^2 + 3y^2)\left((x^2 + 2y^2)^2 - 4x^2y^2\right)$

$= (x^2 - 3y^2)(x^2 + 3y^2)$
$(x^2 + 2y^2 - 2xy)(x^2 + 2y^2 + 2xy)$

$= (x^2 - 3y^2)(x^3 + 3y^2)(x^2 - 2xy + 2y^2)$
$(x^2 + 2xy + 2y^2)$

4 $(81x^4 + 64y^4)(x^4 - y^4)$

$= (81x^4 + 144x^2y^2 + 64y^4 - 144x^2y^2)$
$(x^2 - y^2)(x^2 + y^2)$

$= \left((9x^2 + 8y^2)^2 - 144x^2y^2\right)(x - y)(x + y)(x^2 + y^2)$

$= (9x^2 + 8y^2 - 12xy)(9x^2 + 8y^2 + 12xy)$
$(x - y)(x + y)(x^2 + y^2)$

$= (9x^2 - 12xy + 8y^2)(9x^2 + 12xy + 8y^2)$
$(x - y)(x + y)(x^2 + y^2)$

Answers of General Exercises on factorization

1 $(5x - 3y)(5x + 3y)$

2 $2x^2(x^3 + 27) = 2x^2(x + 3)(x^2 - 3x + 9)$

3 $(2y + 3)(y + 1)$

4 $2(x^4 - 9) = 2(x^2 - 3)(x^2 + 3)$

5 $2(x^2 - 10x + 24) = 2(x - 6)(x - 4)$

6 $(x + 4)^2$

7 $(2x + 3)(4x^2 - 6x + 9)$

8 $(y - 51)(y + 1)$

9 $(5x - 3)^2$

10 $(x - 9)(x + 9)$

11 $y(y^4 - 1) = y(y^2 - 1)(y^2 + 1)$

$= y(y - 1)(y + 1)(y^2 + 1)$

12 $(3x - 2)(x + 3)$

13 $(x - 6)(x - 2)$

14 $3x(x^2 + 4) + 2(x^2 + 4) = (x^2 + 4)(3x + 2)$

15 $(x - 5)(x^2 + 5x + 25)$

16 $(2x - 3)^2$

17 $a^2(a + 3) - 9(a + 3)$

$= (a^2 - 9)(a + 3) = (a - 3)(a + 3)(a + 3)$

18 $(x + 2)^3 - 4(x + 2) = (x + 2)\left((x + 2)^2 - 4\right)$

$= (x + 2)(x + 2 - 2)(x + 2 + 2) = x(x + 2)(x + 4)$

19 $-(2x^2 + 15x + 7) = -(2x + 1)(x + 7)$

20 $(x - 5)(x - 2)$

21 $4x^4 + 4x^2y^2 + y^4 - 4x^2y^2 = (2x^2 + y^2)^2 - 4x^2y^2$

$= (2x^2 + y^2 - 2xy)(2x^2 + y^2 + 2xy)$

$= (2x^2 - 2xy + y^2)(2x^2 + 2xy + y^2)$

22 $(3x^2 - 4y^2)(3x^2 + 4y^2)$

23 $(x^2 - 4)(x^2 - 5) = (x - 2)(x + 2)(x^2 - 5)$

24 $(1 - 2x)(1 + 2x)$

25 $(a^3 - 25b^3)(a^3 + 25b^3)$

26 $\left((x + y) - x\right)\left((x + y)^2 + x(x + y) + x^2\right)$

$= y(x^2 + 2xy + y^2 + x^2 + xy + x^2)$

$= y(3x^2 + 3xy + y^2)$

27 $(7x + 5y^2)^2$

28 $(5x + 2)(x - 1)$

29 $x^4 - 2x^2y^2 + y^4 - 11x^2y^2 + 2x^2y^2$

$= (x^2 - y^2)^2 - 9x^2y^2$

$= (x^2 - y^2 - 3xy)(x^2 - y^2 + 3xy)$

$= (x^2 - 3xy - y^2)(x^2 + 3xy - y^2)$

30 $3x^2(x^2-5x+4)=3x^2(x-4)(x-1)$

31 $(3x-1)(x-6)$

32 $(2x+7y)^2$

33 $(x^3-8y^3)(x^3+8y^3)$

$=(x-2y)(x^2+2xy+4y^2)(x+2y)$
$\times(x^2-2xy+4y^2)$

34 $2y^3(y-2)+7(y-2)=(y-2)(2y^3+7)$

35 $3(5a^4-2a^2b-7b^2)=3(5a^2-7b)(a^2+b)$

36 $6x^2-7xy+2y^2=(2x-y)(3x-2y)$

37 $64x^4+16x^2y^2+y^4-16x^2y^2$

$=(8x^2+y^2)^2-16x^2y^2$

$=(8x^2+y^2-4xy)(8x^2+y^2+4xy)$

$=(8x^2-4xy+y^2)(8x^2+4xy+y^2)$

38 $(x^2+3)(x^2-8)$

39 $5(4x^4+8x^2y^2+9y^4)$

$=5(4x^4+12x^2y^2+9y^4+8x^2y^2-12x^2y^2)$

$=5((2x^2+3y^2)^2-4x^2y^2)$

$=5(2x^2+3y^2-2xy)(2x^2+3y^2+2xy)$

$=5(2x^2-2xy+3y^2)(2x^2+2xy+3y^2)$

40 $(9x^2-4y^2)(x^2-y^2)$

$=(3x-2y)(3x+2y)(x-y)(x+y)$

Answers of Exercise 8

1

1 $x(x-6)=0$ $\therefore x=0$ or $x-6=0$

$\therefore x=6$ $S.S=\{0,6\}$

2 $(x-4)(x+4)=0$

$\therefore x-4=0$ then $x=4$ or $x+4=0$ then $x=-4$

$\therefore S.S=\{4,-4\}$

3 $(2x-5)(2x+5)=0$ $\therefore 2x-5=0$

$\therefore x=\frac{5}{2}$ or $2x+5=0$ then $x=\frac{-5}{2}$

$S.S=\{\frac{5}{2},\frac{-5}{2}\}$

4 $(x+3)(x+2)=0$

$\therefore x+3=0$ then $x=-3$ or $x+2=0$ then $x=-2$

$S.S=\{-3,-2\}$

5 $(x-5)(x-3)=0$

$\therefore x-5=0$ then $x=5$ or $x-3=0$ then $x=3$

$S.S=\{5,3\}$

6 $(x+9)(x-2)=0$

$\therefore x+9=0$ then $x=-9$ or $x-2=0$ $\therefore x=2$

$S.S=\{-9,2\}$

7 $(x-5)(x+4)=0$

$\therefore x-5=0$ then $x=5$ or $x+4=0$ then $x=-4$

$S.S=\{5,-4\}$

8 $(3x+1)(2x-3)=0$ $\therefore 3x+1=0$

$\therefore x=-\frac{1}{3}$ or $2x-3=0$ then $x=\frac{3}{2}$

$\therefore S.S=\{-\frac{1}{3},\frac{3}{2}\}$

9 $(2x-1)(x+4)=0$

$\therefore 2x-1=0$ then $x=\frac{1}{2}$ or $x+4=0$ then $x=-4$

$S.S=\{\frac{1}{2},-4\}$

10 $(2x+1)(x-3)=0$ $\therefore 2x+1=0$

$\therefore x=-\frac{1}{2}$ or $x-3=0$ then $x=3$

$S.S=\{-\frac{1}{2},3\}$

11 $(x+2)^2=0$ then $x+2=0$ $\therefore x=-2$

$S.S=\{-2\}$

12 $(3x-1)^2=0$ $\therefore 3x-1=0$

$\therefore x=\frac{1}{3}$ $\therefore S.S=\{\frac{1}{3}\}$

2

1 $x^2-x=0$ $\therefore x(x-1)=0$

$\therefore x=0$ or $x-1=0$ then $x=1$

$\therefore S.S=\{0,1\}$

2 $3x^2-7x=0$ $\therefore x(3x-7)=0$

$\therefore x=0$ or $3x-7=0$ then $x=\frac{7}{3}$

$\therefore S.S=\{0,\frac{7}{3}\}$

3 $4x^2-49=0$ $\therefore (2x-7)(2x+7)=0$

$\therefore (2x-7)=0$ then $x=\frac{7}{2}$ or $(2x+7)=0$

then $x=\frac{-7}{2}$ $S.S=\{\frac{7}{2},-\frac{7}{2}\}$

4 $x^2+x-6=0$ $\therefore (x+3)(x-2)=0$

$\therefore x+3=0$ then $x=-3$ or $x-2=0$ then $x=2$

$\therefore S.S=\{-3,2\}$

5 $x^2-8x+15=0$ $\therefore (x-3)(x-5)=0$

$\therefore x-3=0$ then $x=3$ or $x-5=0$ then $x=5$

$\therefore S.S=\{3,5\}$

6 $x^2-2x-15=0$ $\therefore (x-5)(x+3)=0$

$\therefore x-5=0$ then $x=5$ or $x+3=0$ then $x=-3$

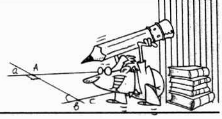
$\therefore S.S=\{5,-3\}$

7 $2x^2-10x+12=0$ $\therefore x^2-5x+6=0$

$\therefore (x-3)(x-2)=0$

$\therefore x-3=0$ then $x=3$ or $x-2=0$ then $x=2$

$S.S=\{3,2\}$



8 $6x^2 - x - 22 = 0 \qquad \therefore (x-2)(6x+11) = 0$

$\therefore x - 2 = 0$ then $x = 2$ or $6x + 11 = 0$ then $x = \frac{-11}{6}$

$S.S = \{2, \frac{-11}{6}\}$

9 $5x^2 + 12x - 44 = 0 \qquad \therefore (5x+22)(x-2) = 0$

$\therefore 5x + 22 = 0$ then $x = \frac{-22}{5}$ or $x - 2 = 0$ then $x = 2$

$S.S = \{\frac{-22}{5}, 2\}$

10 $12x^2 - 47x + 45 = 0 \qquad \therefore (4x-9)(3x-5) = 0$

$\therefore 4x - 9 = 0$ then $x = \frac{9}{4}$ or $3x - 5 = 0$ then $x = \frac{5}{3}$

$S.S = \{\frac{9}{4}, \frac{5}{3}\}$

11 $x^2 + 3 = 12 \qquad \therefore x^2 - 9 = 0$

$\therefore (x-3)(x+3) = 0$

$\therefore x - 3 = 0$ then $x = 3$ or $x + 3 = 0$ then $x = -3$

$\therefore S.S = \{3, -3\}$

12 $x^2 - 3x - 5x = 0 \qquad \therefore x^2 - 8x = 0$

$\therefore x(x-8) = 0$

$\therefore x = 0$ or $x - 8 = 0$ then $x = 8$

$S.S = \{0, 8\}$

3

1 $x^2 - 5x + 6 = 0 \qquad \therefore (x-3)(x-2) = 0$

$\therefore x - 3 = 0$ then $x = 3$ or $x - 2 = 0$ then $x = 2$

$S.S = \{3, 2\}$

2 $x^2 + 3x - 10 = 0 \qquad \therefore (x+5)(x-2) = 0$

$\therefore x + 5 = 0$ then $x = -5$ or $x - 2 = 0$ then $x = 2$

$S.S = \{-5, 2\}$

3 $x^2 + 2x - 8 + 5 = 0 \qquad \therefore x^2 + 2x - 3 = 0$

$\therefore (x+3)(x-1) = 0$

$\therefore x + 3 = 0$ then $x = -3$ or $x - 1 = 0$ then $x = 1$

$S.S = \{-3, 1\}$

4 $x^2 - 2x - 3 - 5 = 0 \qquad \therefore x^2 - 2x - 8 = 0$

$\therefore (x-4)(x+2) = 0$

$\therefore x - 4 = 0$ then $x = 4$ or $x + 2 = 0$ then $x = -2$

$S.S = \{4, -2\}$

5 $x^2 + 5x - 24 - 3x = 0 \qquad \therefore x^2 + 2x - 24 = 0$

$\therefore (x+6)(x-4) = 0$

$\therefore x + 6 = 0$ then $x = -6$ or $x - 4 = 0$ then $x = 4$

$S.S = \{-6, 4\}$

6 $2x^2 - 10x - 20 + 4x = 0$

$\therefore 2x^2 - 6x - 20 = 0 \qquad \therefore x^2 - 3x - 10 = 0$

$\therefore (x-5)(x+2) = 0$

$\therefore x - 5 = 0$ then $x = 5$ or $x + 2 = 0$ then $x = -2$

$S.S = \{5, -2\}$

7 $(x+3-7)(x+3+7) = 0$

$\therefore (x-4)(x+10) = 0$

$\therefore x - 4 = 0$ then $x = 4$ or $x + 10 = 0$ then $x = -10$

$S.S = \{4, -10\}$

8 $(x-2)^2 - 81 = 0$

$\therefore (x-2-9)(x-2+9) = 0$

$\therefore (x-11)(x+7) = 0$

$\therefore x - 11 = 0$ then $x = 11$ or $x + 7 = 0$ then $x = -7$

$S.S = \{11, -7\}$

9 $4(x+5)^2 - 25 = 0$

$\therefore (2(x+5)-5)(2(x+5)+5) = 0$

$\therefore (2x+5)(2x+15) = 0$

$\therefore 2x + 5 = 0$ then $x = \frac{-5}{2}$

or $2x + 15 = 0$ then $x = \frac{-15}{2}$

$\therefore S.S = \{\frac{-5}{2}, \frac{-15}{2}\}$

10 $x^2 - 2x + 1 + x - 3 = 0 \quad \therefore (x^2 - x - 2) = 0$

$\therefore (x-2)(x+1) = 0$

$\therefore x - 2 = 0$ then $x = 2$ or $x + 1 = 0$ then $x = -1$

$S.S = \{2, -1\}$

11 $(x+3)(2(x+3)+7) = 0$

$\therefore (x+3)(2x+13) = 0$

$\therefore x + 3 = 0$ then $x = -3$

or $2x + 13 = 0$ then $x = \frac{-13}{2}$

$S.S = \{-3, \frac{-13}{2}\}$

12 $4x^2 + 4x + 1 = 9x^2 - 6x + 1$

$\therefore 4x^2 - 9x^2 + 4x + 6x + 1 - 1 = 0$

$\therefore -5x^2 + 10x = 0 \qquad \therefore x^2 - 2x = 0$

$\therefore x(x-2) = 0$

$\therefore x = 0$ or $x - 2 = 0$ then $x = 2$

$S.S = \{0, 2\}$

13 $4x^2 - 4x + 1 + x^2 - 2x + 1 - 10 = 0$

$\therefore 5x^2 - 6x - 8 = 0 \qquad \therefore (5x+4)(x-2) = 0$

$\therefore 5x + 4 = 0$ then $x = \frac{-4}{5}$ or $x - 2 = 0$ then $x = 2$

$S.S = \{\frac{-4}{5}, 2\}$

14 $x^2 + 6x + 9 + 3x + 9 - 10 = 0$

$\therefore x^2 + 9x + 8 = 0 \qquad \therefore (x+1)(x+8) = 0$

$\therefore x + 1 = 0$ then $x = -1$ or $x + 8 = 0$ then $x = -8$

$S.S = \{-1, -8\}$

4

1 $2x(x^2-4)=0$ $\therefore 2x(x-2)(x+2)=0$

$\therefore 2x=0$ then $x=0$ or $x-2=0$ then $x=2$

or $x+2=0$ then $x=-2$

S.S $=\{0, 2, -2\}$

2 $4x^3-9x=0$ $\therefore x(4x^2-9)=0$

$\therefore x(2x-3)(2x+3)=0$

$\therefore x=0$ or $2x-3=0$ then $x=\frac{3}{2}$

or $2x+3=0$ then $x=\frac{-3}{2}$

S.S $=\{0, \frac{3}{2}, \frac{-3}{2}\}$

3 $(x^2-4)(x^2-1)=0$

$\therefore (x-2)(x+2)(x-1)(x+1)=0$

$\therefore x-2=0$ then $x=2$ or $x+2=0$ then $x=-2$

or $x-1=0$ then $x=1$ or $x+1=0$ then $x=-1$

S.S $=\{2, -2, 1, -1\}$

4 $x^4-26x^2+25=0$

$\therefore (x^2-25)(x^2-1)=0$

$\therefore (x-5)(x+5)(x-1)(x+1)=0$

$\therefore x-5=0$ then $x=5$ or $x+5=0$ then $x=-5$

or $x-1=0$ then $x=1$ or $x+1=0$ then $x=-1$

S.S $=\{5, -5, 1, -1\}$

5 $(x^2-4)(x^2+4)=0$

$\therefore (x-2)(x+2)(x^2+4)=0$

$\therefore x-2=0$ then $x=2$

or $x+2=0$ then $x=-2$

or $x^2+4=0$ (has no solution in $\mathbb{R}$)

$\therefore$ The S.S. $=\{2, -2\}$

5

1 Multiplying the equation by 3

$\therefore 3y^2-7y+4=0$ $\therefore (3y-4)(y-1)=0$

$\therefore 3y-4=0$ then $y=\frac{4}{3}$ or $y-1=0$ then $y=1$

S.S $=\{\frac{4}{3}, 1\}$

2 Multiplying the equation by 2

$\therefore 2x^2-2x-3-9=0$ $\therefore 2x^2-2x-12=0$

$\therefore x^2-x-6=0$ $\therefore (x-3)(x+2)=0$

$\therefore x-3=0$ then $x=3$ or $x+2=0$ then $x=-2$

S.S $=\{3, -2\}$

3 Multiplying the equation by x

$\therefore x^2+2=3x$ $\therefore x^2-3x+2=0$

$\therefore (x-2)(x-1)=0$

$\therefore x-2=0$ then $x=2$ or $x-1=0$ then $x=1$

S.S $=\{2, 1\}$

4 Multiplying the equation by $2x$

$\therefore 2x^2-10=x$ $\therefore 2x^2-x-10=0$

$\therefore (2x-5)(x+2)=0$

$\therefore 2x-5=0$ then $x=\frac{5}{2}$ or $x+2=0$ then $x=-2$

S.S $=\{\frac{5}{2}, -2\}$

5 Multiplying the equation by $5x$

$\therefore x(x-1)=30$ $\therefore x^2-x-30=0$

$\therefore (x+5)(x-6)=0$ $\therefore x+5=0$, then $x=-5$

or $x-6=0$, then $x=6$

$\therefore$ The S.S. $=\{-5, 6\}$

6

1 c 2 c 3 d 4 d

5 a 6 d 7 b 8 c

7

1 3 2 8, 4 3 0, 4 4 $\{4, -\frac{1}{2}\}$

8

Multiplying both sides of the equation by x

$\therefore x^2+1=2x$ $\therefore x^2-2x+1=0$

$\therefore (x-1)^2=0$ $\therefore x=1$

$\therefore x^2+\frac{1}{x^2}=1^2+\frac{1}{1^2}=2$

Another solution :

Squaring the two sides of the equation

$\therefore \left(x+\frac{1}{x}\right)^2=4$ $\therefore x^2+2+\frac{1}{x^2}=4$

$\therefore x^2+\frac{1}{x^2}=4-2=2$

9

$\because x^2+\frac{1}{x^2}=34$,

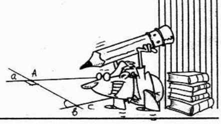
Adding 2 to the both sides

$\therefore x^2+2+\frac{1}{x^2}=34+2$

$\therefore \left(x+\frac{1}{x}\right)^2=36$,

Taking the square root to both sides

$\therefore x+\frac{1}{x}=\pm\sqrt{36}=\pm 6$



Another solution :

$\left(x+\frac{1}{x}\right)^2 = x^2+2+\frac{1}{x^2} = x^2+\frac{1}{x^2}+2$

$= 34 + 2 = 36$

Taking the square root to both sides

$\therefore x+\frac{1}{x} = \pm\sqrt{36} = \pm 6$

10

Multiply the equation by 12

$\therefore 2x(x-2) - 3x(x+1) + 28(x-3) - 24 = 0$

$\therefore 2x^2 - 4x - 3x^2 - 3x + 28x - 84 - 24 = 0$

$\therefore -x^2 + 21x - 108 = 0$ $\quad \therefore x^2 - 21x + 108 = 0$

$\therefore (x-9)(x-12) = 0$ $\quad \therefore x - 9 = 0$ then $x = 9$

or $x - 12 = 0$ then $x = 12$ $\quad$ S.S $= \{9 , 12\}$

11

$\because$ 2 , 3 are the two roots of the equation

$\therefore$ the equation is $(x-2)(x-3) = 0$

i.e. $x^2 - 5x + 6 = 0$ $\quad \therefore b = 5 , c = 6$

Answers of Exercise 9

1

1 c $\quad$ 2 c $\quad$ 3 b $\quad$ 4 c

5 d $\quad$ 6 d $\quad$ 7 c

2

Let the number be x $\quad \therefore x^2 - 5x = 36$

$\therefore x^2 - 5x - 36 = 0$ $\quad (x+4)(x-9) = 0$

$\therefore x + 4 = 0$, then $x = -4$ (refused)

or $x - 9 = 0$, then $x = 9$ $\quad \therefore$ The number is 9

3

Let the number be x

$\therefore 2x^2 + 7 = 135$ $\quad \therefore 2x^2 - 128 = 0$

$\therefore x^2 - 64 = 0$ $\quad \therefore (x-8)(x+8) = 0$

$\therefore x - 8 = 0$, then $x = 8$ or $x + 8 = 0$, then $x = -8$

$\therefore$ The number is 8 or -8

4

Let the number be x $\quad \therefore 4x^2 = 81$

$\therefore 4x^2 - 81 = 0$ $\quad \therefore (2x-9)(2x+9) = 0$

$\therefore 2x - 9 = 0$, then $x = \frac{9}{2}$,

or $2x + 9 = 0$, then $x = \frac{-9}{2}$

$\therefore$ The number is $\frac{9}{2}$ or $-\frac{9}{2}$

5

Let the number be x $\quad \therefore x^2 = 6x$

$\therefore x^2 - 6x = 0$ $\quad \therefore x(x-6) = 0$

$\therefore x = 0$ (refused) or $x - 6 = 0$, then $x = 6$

$\therefore$ The number is 6

6

Let the number be x

$\therefore x + x^2 = 12$ $\quad \therefore x^2 + x - 12 = 0$

$\therefore (x+4)(x-3) = 0$

$\because x + 4 = 0$, then $x = -4$ or $x - 3 = 0$, then $x = 3$

$\therefore$ The number is -4 or 3

7

Let the number be x $\quad \therefore x^2 - 2x = 48$

$\therefore x^2 - 2x - 48 = 0$ $\quad \therefore (x+6)(x-8) = 0$

$\therefore x + 6 = 0$, then $x = -6$ (refused)

or $x - 8 = 0$, then $x = 8$

$\therefore$ The number is 8

8

Let the first number be x

$\therefore$ The second number $= 20 - x$

$x(20-x) = 75$ $\quad \therefore 20x - x^2 - 75 = 0$

$\therefore x^2 - 20x + 75 = 0$ $\quad \therefore (x-15)(x-5) = 0$

$\therefore x - 15 = 0$, then $x = 15$

or $x - 5 = 0$, then $x = 5$

$\therefore$ The numbers are 5 and 15

9

Let the first number be x

$\therefore$ The second number $= x + 5$

$\therefore x^2 + (x+5)^2 = 73$

$\therefore x^2 + x^2 + 10x + 25 - 73 = 0$

$\therefore 2x^2 + 10x - 48 = 0$ $\quad \therefore x^2 + 5x - 24 = 0$

$(x+8)(x-3) = 0$

$\therefore x + 8 = 0$, then $x = -8$

i.e. The two numbers are -8 and -3

or $x - 3 = 0$ then $x = 3$

i.e. The two numbers are 3 and 8

10

Let the first number be x

$\therefore$ The second number $= x + 4$

$\therefore x(x+4) = 45$ $\therefore x^2 + 4x - 45 = 0$

$\therefore (x+9)(x-5) = 0$

$\therefore x + 9 = 0$, then $x = -9$

i.e. The two numbers are -9 and -5

or $x - 5 = 0$, then $x = 5$

i.e. The two numbers are 5 and 9

11

Let the first number be x

$\therefore$ The second number $= x + 2$

$\therefore x^2 + (x+2)^2 = 130$

$\therefore x^2 + x^2 + 4x + 4 - 130 = 0$

$\therefore 2x^2 + 4x - 126 = 0$

$\therefore x^2 + 2x - 63 = 0$ $\therefore (x+9)(x-7) = 0$

$\therefore x + 9 = 0$, then $x = -9$

i.e. The two numbers are -9 and -7

or $x - 7 = 0$ then , $x = 7$

i.e. The two numbers are 7 and 9

12

Let the numbers be x , $x + 1$, $x + 2$

$\therefore x + x + 1 + x + 2 = (x+1)^2$

$\therefore 3x + 3 = x^2 + 2x + 1$ $\therefore x^2 - x - 2 = 0$

$\therefore (x-2)(x+1) = 0$ $\therefore x - 2 = 0$, then $x = 2$

i.e. The three numbers are 2 , 3 and 4

or $x + 1 = 0$, then $x = -1$

i.e. The three numbers are -1 , 0 and 1

13

Let the first number be $7x$

$\therefore$ The second number is $8x$

$\therefore (7x) \times (8x) - 9(8x) = 80$

$\therefore 56x^2 - 72x - 80 = 0$

$\therefore 7x^2 - 9x - 10 = 0$ $(7x+5)(x-2) = 0$

$\therefore 7x + 5 = 0$, then $x = -\frac{5}{7}$ (refused)

or $x - 2 = 0$, then $x = 2$

i.e. The two numbers are 14 and 16

14

Let the first number be x

$\therefore 2x^2 + (-x) = 91$ $\therefore 2x^2 - x - 91 = 0$

$\therefore (2x+13)(x-7) = 0$ $\therefore 2x + 13 = 0$

, then $x = \frac{-13}{2}$ (refused) or $x - 7 = 0$, then $x = 7$

$\therefore$ The number is 7

15

Let the number be x

$\therefore x - \frac{1}{x} = \frac{5}{6}$ multiplying by $6x$

$\therefore 6x^2 - 6 = 5x$

$\therefore 6x^2 - 5x - 6 = 0$ $\therefore (2x-3)(3x+2) = 0$

$\therefore 2x - 3 = 0$, then $x = \frac{3}{2}$ or $(3x+2) = 0$

, then $x = \frac{-2}{3}$

$\therefore$ The number is $\frac{3}{2}$ or $\frac{-2}{3}$

16

Let the tens digit be x

$\therefore$ the units digit is $2x$ $\therefore x(2x) - (x + 2x) = 9$

$\therefore 2x^2 - 3x - 9 = 0$ $\therefore (2x+3)(x-3) = 0$

$\therefore 2x + 3 = 0$, then $x = \frac{-3}{2}$ (refused)

or $x - 3 = 0$, then $x = 3$ $\therefore$ The number is 36

17

Let the age of Said now be x years

$\therefore x^2 - 3(x-4) = 192$ $\therefore x^2 - 3x + 12 - 192 = 0$

$\therefore x^2 - 3x - 180 = 0$ $\therefore (x-15)(x+12) = 0$

$\therefore x - 15 = 0$, then $x = 15$

or $x + 12 = 0$, then $x = -12$ (refused)

$\therefore$ The age of Said now is 15 years.

18

Let the age of Hatem now be x years

$\therefore$ The age of Hanan now $= (x - 4)$ years

$\therefore x^2 + (x-4)^2 = 26$

$\therefore x^2 + x^2 - 8x + 16 - 26 = 0$

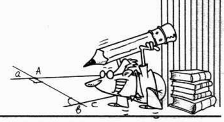
$\therefore 2x^2 - 8x - 10 = 0$ $\therefore x^2 - 4x - 5 = 0$

$\therefore (x-5)(x+1) = 0$ $\therefore x - 5 = 0$, then $x = 5$

or $x + 1 = 0$, then $x = -1$ (refused)

$\therefore$ The age of Hatem = 5 years and

The age of Hanan = one year.



19

Let the age of Anees now be X years

$\therefore$ the age of Kamal now = $(X + 3)$ years

since 4 years

The age of Anees = $(X - 4)$ years

The age of Kamal = $(X + 3 - 4) = (X - 1)$ years.

$\therefore (X - 4)(X - 1) = 18 \qquad \therefore X^2 - 5X + 4 - 18 = 0$

$\therefore X^2 - 5X - 14 = 0 \qquad \therefore (X + 2)(X - 7) = 0$

$\therefore X + 2 = 0$, then $X = -2$ (refused)

or $X - 7 = 0$, then $X = 7$

$\therefore$ The age of Anees now is 7 years.

The age of Kamal now is 10 years.

20

Let the width of the rectangle be X cm.

$\therefore$ The length of the rectangle is $(X + 4)$ cm.

$\therefore X(X + 4) = 21 \qquad \therefore X^2 + 4X - 21 = 0$

$\therefore (X + 7)(X - 3) = 0$

$\therefore X + 7 = 0$, then $X = -7$ (refused)

or $X - 3 = 0$, then $X = 3$

$\therefore$ The width = 3 cm. and the length = 7 cm.

21

Let the width of the rectangle be X cm.

The length of the rectangle = $(X + 7.5)$ cm.

$\therefore X(X + 7.5) = 46 \qquad \therefore X^2 + 7.5X - 46 = 0$

$\therefore 2X^2 + 15X - 92 = 0 \qquad \therefore (2X + 23)(X - 4) = 0$

$\therefore 2X + 23 = 0$, then $X = -\frac{23}{2}$ (refused)

or $X - 4 = 0$, then $X = 4$

$\therefore$ The width = 4 cm. The length = 11.5 cm.

The perimeter = 2 (4 + 11.5) = 31 cm.

22

Let the width of the rectangle be X cm.

$\therefore$ The length of the rectangle is $(X + 5)$ cm.

The area of the rectangle = $X(X + 5)$ cm^2.

The side length of the square = $(3X)$ cm.

The area of the square = $9X^2$ cm^2.

$9X^2 - X(X + 5) = 57$

$\therefore 9X^2 - X^2 - 5X - 57 = 0$

$\therefore 8X^2 - 5X - 57 = 0$

$(X - 3)(8X + 19) = 0 \qquad \therefore X - 3 = 0$

, then $X = 3$ or $8X + 19 = 0$

, then $X = -\frac{19}{8}$ (refused)

$\therefore$ The width of the rectangle = 3 cm.

and the length = 8 cm.

and the side length of the square = 9 cm.

23

m (∠ BCD) + m (∠ ACD) = 180°

$\therefore X^2 + 8X = 180° \qquad \therefore X^2 + 8X - 180° = 0$

$\therefore (X - 10°)(X + 18°) = 0 \quad \therefore X - 10° = 0$, then $X = 10°$

or $X + 18° = 0$, then $X = -18°$ (refused)

24

$\because$ m (∠ A) + m (∠ B) + m (∠ C) = 180°

$\therefore X^2 + 61° + 110° - 11X + 90° - 7X = 180°$

$\therefore X^2 - 18X + 261° - 180° = 0$

$\therefore X^2 - 18X + 81° = 0 \quad \therefore (X - 9°)^2 = 0 \quad \therefore X = 9°$

$\therefore$ m (∠ A) = 142° , m (∠ B) = 11° and m (∠ C) = 27°

25

Let the length of one side of the right angle be X cm.

$\therefore$ The length of the other side = $(X - 2)$ cm.

$\frac{1}{2}X(X - 2) = 24$ multiplying by 2

$\therefore X(X - 2) = 48 \qquad \therefore X^2 - 2X - 48 = 0$

$(X - 8)(X + 6) = 0 \qquad \therefore X - 8 = 0$, then $X = 8$

or $X + 6 = 0$, then $X = -6$ (refused)

$\therefore$ The two lengths of the sides of the right angle are 8 cm. and 6 cm.

26

$\because$ The area of the triangle = 24 cm^2.

$\therefore \frac{1}{2}(5X + 3)(X + 5) = 24$

$\therefore (5X + 3)(X + 5) = 48$

$\therefore 5X^2 + 28X + 15 - 48 = 0$

$5X^2 + 28X - 33 = 0 \qquad \therefore (5X + 33)(X - 1) = 0$

$\therefore 5X + 33 = 0$, then $X = -\frac{33}{5}$ (refused)

or $X - 1 = 0$, then $X = 1$

$\therefore$ The lengths of the two sides of the right angle are 8 and 6 cm.

$\therefore$ The length of the hypotenuse = 10 cm.

The perimeter of the triangle = 8 + 6 + 10 = 24 cm.

27

$\because$ The triangle is right angled

$\therefore$ The hypotenuse $= (2\,x + 1)$

$\therefore (2\,x)^2 + (x - 11)^2 = (2\,x + 1)^2$

$\therefore 4\,x^2 + x^2 - 22\,x + 121 = 4\,x^2 + 4\,x + 1$

$\therefore x^2 - 26\,x + 120 = 0 \qquad \therefore (x - 20)(x - 6) = 0$

$\therefore x - 20 = 0$, then $x = 20$

or $x - 6 = 0 \qquad \therefore x = 6$ (refused)

$\therefore$ The lengths of the sides of the triangle are 40 cm. , 41 cm. and 9 cm.

$\therefore$ The perimeter of the triangle $= 40 + 41 + 9 = 90$ cm.

The area of the triangle $= \frac{1}{2} \times 40 \times 9 = 180$ cm.^{2}

28

Let the width of the rectangle be x

$\therefore$ The length of the rectangle $= 2\,x$

$\therefore$ The area $= 2\,x^2$

$\therefore (2\,x + 1)(x - 1) = 2\,x^2 - 7$

$\therefore 2\,x^2 - x - 1 = 2\,x^2 - 7$

$\therefore -x + 6 = 0 \qquad \therefore x = 6$

$\therefore$ The width of the rectangle = 6 cm.

The length of the rectangle = 12 cm.

29

$\because \Delta\,MCD \sim \Delta\,MAB$

$\therefore \frac{MC}{MA} = \frac{CD}{AB} = \frac{MD}{MB} \qquad \therefore \frac{3}{MA} = \frac{MD}{4}$

, $\because MD = AD - MA = 7 - MA$

$\therefore \frac{3}{MA} = \frac{7 - MA}{4} \qquad \therefore 12 = 7\,MA - (MA)^2$

$\therefore (MA)^2 - 7\,MA + 12 = 0 \;\therefore (MA - 4)(MA - 3) = 0$

$\therefore MA - 4 = 0$, then $MA = 4$

or $MA - 3 = 0$, then $MA = 3$

(refused because $MA > MC$)

30

$\because x \times 10 + x(13 - x) = 60$

$\therefore 10\,x + 13\,x - x^2 - 60 = 0$

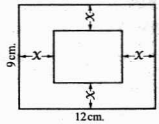
$\therefore x^2 - 23\,x + 60 = 0 \qquad (x - 3)(x - 20) = 0$

$\therefore x - 3 = 0$, then $x = 3$

or $x - 20 = 0$, then $x = 20$ (refused)

(where $x < 13$)

31



Let the width of the tape be x metre as shown in the figure

$\therefore$ the width of the carbet

$= (9 - 2\,x)$ metre.

The length of the carbet $= (12 - 2\,x)$ metre.

The area of the carbet $= (12 - 2\,x)(9 - 2\,x)$

$= \frac{1}{2}$ the area of the room

$\therefore (12 - 2\,x)(9 - 2\,x) = \frac{1}{2} \times 9 \times 12$

$\therefore 108 - 42\,x + 4\,x^2 = 54 \qquad \therefore 4\,x^2 - 42\,x + 54 = 0$

$\therefore 2\,x^2 - 21\,x + 27 = 0 \qquad \therefore (2\,x - 3)(x - 9) = 0$

$\therefore 2\,x - 3 = 0$, then $x = \frac{3}{2}$ or $x - 9 = 0$, then $x = 9$ (refused) because it equals the width of the room.

$\therefore$ The width of the tape = 1.5 metre.

Answers of the general exercises on unit one

Completion questions

1 $2\,a^2$, 6

2 5 , $2\,x^2$, $7\,x$

3 x , $7\,x\,y$, $6\,y^2$

4 b , a , 7 a b

5 $7\,x$, 5 y , $70\,x\,y$

6 9 , $7\,x$, $7\,x$

7 8 , 2 , x^2 , $2\,x$

8 1 , 2

9 1

10 2 a , 2 a b , b^2

11 $7\,y^2$, $11\,x$, 7 y

12 $12\,x\,y$, $9\,y^2$, $2\,x$

13 3 , $25\,a^2$, 9

14 x , y

15 2 , $x + 4$

16 14 , -14 , 2 , -2

17 -14 , -2

18 1

19 ± 8

20 19

21 -42

22 $x + 5$

23 $x^2 - 5\,x + 25$

24 3

25 $(y - z)(x + l)$

26 $\varnothing$

27 $\{1 , -1\}$

28 1

29 $\frac{1}{4}\,x^2$ cm.^{2}

30 $(x + 5)$ years

Multiple choice questions

1 c	2 b	3 d	4 a	5 a	6 b
7 a	8 b	9 c	10 b	11 d	12 b
13 a	14 b	15 b	16 c	17 c	18 c
19 b	20 c	21 c	22 d	23 d	24 d
25 c	26 c	27 a	28 c	29 c	30 c

Answers of Unit Two

Answers of Exercise 10

1

1. $\frac{1}{3^2} = \frac{1}{9}$
2. 4
3. $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$
4. $5^2 = 25$
5. $\left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$
6. $\left(-\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$
7. $\left(\frac{1}{\sqrt[3]{5}}\right)^3 = \frac{1}{5}$
8. $\left(\sqrt{5}\right)^2 = 5$
9. $\left(\frac{1}{100}\right)^{-2} = (100)^2 = 10000$
10. $\left(\frac{2}{10}\right)^{-2} = \left(\frac{1}{5}\right)^{-2} = 5^2 = 25$
11. $\left(\frac{1}{\sqrt{2}}\right)^3 = \frac{1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$
12. $\left(\frac{3}{\sqrt{3}}\right)^5 = \left(\frac{\sqrt{3}\times\sqrt{3}}{\sqrt{3}}\right)^5 = \left(\sqrt{3}\right)^5 = 9\sqrt{3}$

2

1. $x^{\text{zero}} = 1$
2. $x^{-4+3} = x^{-1} = \frac{1}{x}$
3. $x^{-6} \times x^{6} = x^{\text{zero}} = 1$
4. $x^{2-3+4-1} = x^2$
5. $x^{-6-2+3+4} = x^{-1} = \frac{1}{x}$

3

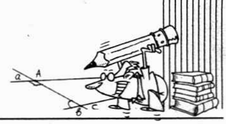
1. $\left(\sqrt{2}\right)^6 = 2^3 = 8$
2. $\left(\sqrt{7}\right)^{5-2-1} = \left(\sqrt{7}\right)^2 = 7$
3. $\left(\sqrt{2}\right)^4 \times \left(\sqrt{2}\right)^2 \times \left(\sqrt{2}\right)^{-2} = \left(\sqrt{2}\right)^{4+2-2}$
   $= \left(\sqrt{2}\right)^4 = 4$
4. $-\left(\sqrt{3}\right) \times \left(\sqrt{3}\right)^3 \times \left(\sqrt{3}\right)^4 = -\left(\sqrt{3}\right)^8 = -3^4 = -81$
5. $\left(\sqrt{5}\right)^{-4+6} = \left(\sqrt{5}\right)^2 = 5$
6. $\left(-\sqrt{5}\right)^4 = 5^2 = 25$
7. $\left(\frac{1}{\sqrt{2}}\right)^6 = \frac{1}{\left(\sqrt{2}\right)^6} = \frac{1}{2^3} = \frac{1}{8}$
8. $\left(\sqrt{2}\right)^6 \times \left(-\sqrt{2}\right)^4 = 2^3 \times 2^2 = 2^5 = 32$
9. $\left(\frac{1}{\sqrt{3}}\right)^4 \times \left(\sqrt{2}\right)^4 = \frac{1}{9} \times 4 = \frac{4}{9}$
10. $(-5)^6 \times \left(-\frac{1}{\sqrt{5}}\right)^4 = 5^6 \times \left(\frac{1}{\sqrt{5}}\right)^4 = 5^6 \times \frac{1}{5^2}$
    $= 5^{6-2} = 5^4 = 625$

4

1. $\left(\sqrt{7}\right)^{-4-3+9} = \left(\sqrt{7}\right)^2 = 7$
2. $\left(\sqrt{3}\right)^{7+8-6} = \left(\sqrt{3}\right)^9 = 81\sqrt{3}$
3. $\left(\sqrt{3}\right)^{8+6-12} = \left(\sqrt{3}\right)^2 = 3$
4. $-\left(\sqrt{5}\right)^{10+5-11} = -\left(\sqrt{5}\right)^4 = -5^2 = -25$
5. $\frac{2^{-2} \times \cancel{\left(\sqrt{7}\right)^{-2}} \times 2^{-2}}{\cancel{\left(\sqrt{7}\right)^{-2}}} = 2^{-4} = \frac{1}{16}$
6. $\frac{3^4 \times \left(\sqrt{2}\right)^4 \times \left(\sqrt{2}\right)^2}{2^2 \times \left(\sqrt{3}\right)^2} = \frac{3^4 \times \left(\sqrt{2}\right)^6}{2^2 \times 3} = \frac{3^4 \times 2^3}{2^2 \times 3}$
   $= 3^{4-1} \times 2^{3-2} = 3^3 \times 2 = 54$
7. $\frac{\left(\sqrt{3}\right)^{-4} \times \left(\sqrt{2}\right)^3 \times 3^5 \times \left(\sqrt{3}\right)^5}{3^5 \times \left(\sqrt{2}\right)^5 \times \sqrt{3}}$
   $= \left(\sqrt{3}\right)^{-4+5-1} \times \left(\sqrt{2}\right)^{3-5}$
   $= \left(\sqrt{3}\right)^{\text{zero}} \times \left(\sqrt{2}\right)^{-2} = \frac{1}{2}$
8. $\frac{\left(\sqrt{3}\right)^5 \times \left(\sqrt{3}\right)^4}{\left(\sqrt{3}\right)^3 \times \left(\sqrt{3}\right)^6} = \left(\sqrt{3}\right)^{5+4-3-6} = \left(\sqrt{3}\right)^{\text{zero}} = 1$
9. $\frac{\left(\sqrt{2}\right)^5 \times 2^6 \times 5^6}{\left(\sqrt{2}\right)^3 \times 2^3 \times 5^5} = \left(\sqrt{2}\right)^{5-3} \times 2^{6-3} \times (5)^{6-5}$
   $= \left(\sqrt{2}\right)^2 \times 2^3 \times 5 = 2 \times 2^3 \times 5 = 2^4 \times 5$
   $= 16 \times 5 = 80$
10. $\frac{3^{-2} \times 5^{-2} \times \left(\sqrt{5}\right)^3 \times 3^3}{3^2 \times \left(\sqrt{5}\right)^{-3}} = 3^{-2+3-2} \times 5^{-2} \times \left(\sqrt{5}\right)^{3+3}$
    $= 3^{-1} \times 5^{-2} \times \left(\sqrt{5}\right)^6 = \frac{1}{3} \times \frac{1}{5^2} \times 5^3 = \frac{1}{3} \times 5^{3-2} = \frac{5}{3}$
11. $\frac{\left(\sqrt{5}\right)^6 \times \left(\sqrt{2}\right)^8}{\left(\sqrt{5 \times 2}\right)^4} = \frac{\left(\sqrt{5}\right)^6 \times \left(\sqrt{2}\right)^8}{\left(\sqrt{5}\right)^4 \times \left(\sqrt{2}\right)^4}$
    $= \left(\sqrt{5}\right)^{6-4} \times \left(\sqrt{2}\right)^{8-4} = \left(\sqrt{5}\right)^2 \times \left(\sqrt{2}\right)^4 = 5 \times 4 = 20$
12. $\frac{(10)^2 \times (10)^{-7}}{(10)^{-2} \times (10)^{-3}} = (10)^{2-7+2+3} = (10)^{\text{zero}} = 1$

5

1. $\frac{2^4 \times \left(\sqrt{3}\right)^4}{\left(\sqrt{2}\right)^4} = \frac{2^4 \times 3^2}{2^2} = 2^2 \times 3^2 = 36$



2 $\frac{3^4\times(\sqrt{2})^4}{2^4\times(\sqrt{3})^4}=\frac{3^4\times2^2}{2^4\times3^2}=\frac{3^2}{2^2}=\frac{9}{4}$

3 $\frac{(-5)^4\times(\sqrt{2})^4}{2^4\times(\sqrt{5})^4}=\frac{5^4\times2^2}{2^4\times5^2}=\frac{5^2}{2^2}=\frac{25}{4}$

4 $\frac{(\sqrt{3})^2}{(\sqrt{2})^2}\times\frac{(\sqrt{2})^4}{(\sqrt{3})^4}=\frac{(\sqrt{2})^2}{(\sqrt{3})^2}=\frac{2}{3}$

6

1 $\frac{3^{2x}\times3^{x+2}}{3^{3x}}=3^{2x+x+2-3x}=3^2=9$

2 $\frac{2^{2x}\times3^{x-1}}{2^{2x}\times3^{x}}=3^{x-1-x}=3^{-1}=\frac{1}{3}$

3 $\frac{2^{x}\times2^{2x+2}}{2^{3x}}=2^{x+2x+2-3x}=2^2=4$

4 $\frac{(2^2\times3^2)^n\times5^{2n}}{(3\times2\times5)^{2n}}=\frac{2^{2n}\times3^{2n}\times5^{2n}}{3^{2n}\times2^{2n}\times5^{2n}}=1$

5 $\frac{2^{x}\times(7^2)^{x-1}}{(2\times7^2)^{x}}=\frac{2^{x}\times7^{2x-2}}{2^{x}\times7^{2x}}=7^{2x-2-2x}=7^{-2}=\frac{1}{49}$

6 $\frac{2^{2x+4}\times3^{6+2x}}{2^{2x+3}\times3^{2x+3}}=2^{2x+4-2x-3}\times3^{6+2x-2x-3}$

$=2^1\times3^3=54$

7 $\frac{2^{2n}\times2^{2n}\times3^{2n}}{2^{4n}\times3^{2n}}=2^{2n+2n-4n}=2^0=1$

8 $\frac{3^{4x}\times2^{2x}\times3^{2x}}{3^{6x-3}\times2^{2x}}=3^{4x+2x-6x+3}=3^3=27$

9 $\frac{2^{n}\times3^{2n+2}\times2}{2\times3\times2^{n}\times3^{2n}}=3^{2n+2-1-2n}=3$

10 $\frac{3\times(2\times3^2)^{x+1}\times2^{x}}{2\times(2^2\times3^2)^{x}}=\frac{3\times2^{x+1}\times3^{2x+2}\times2^{x}}{2\times2^{2x}\times3^{2x}}$

$=3^{1+2x+2-2x}\times2^{x+1+x-1-2x}=3^3\times2^0=27$

11 $\frac{2^n\times3^n\times(2^2)^{n+\frac{1}{2}}}{(2^3\times3)^n}=\frac{2^n\times3^n\times2^{2n+1}}{2^{3n}\times3^n}=2^{n+2n+1-3n}=2$

12 $\frac{2^{3n-3}\times2^{-5n}}{2^5\times2^{-2n}}=2^{3n-3-5n-5+2n}=2^{-8}=\frac{1}{2^8}=\frac{1}{256}$

13 $\frac{(\sqrt{3})^{n+2}\times3^{n+1}\times5^{n+1}}{(\sqrt{3})^{n}\times3^{n}\times5^{n+2}}$

$=(\sqrt{3})^{n+2-n}\times3^{n+1-n}\times5^{n+1-n-2}$

$=(\sqrt{3})^{2}\times3\times5^{-1}=\frac{9}{5}$

14 $\frac{2^{2x+2}\times3^{4-2x}}{2^{2x}\times3^{2x}}=2^{2x+2-2x}\times3^{4-2x-2x}$

$=2^2\times3^{4-4x}$ at $x=1$ $\therefore 2^2\times3^{4-4}=2^2\times3^0=4$

15 $\frac{3^{2x-2}\times2^{3x}}{2^{3x}\times3^{x}}=3^{2x-2-x}\times2^{3x-3x}$

$=3^{x-2}\times2^0=3^{x-2}$ when $x=2$

$\therefore 3^{2-2}=3^0=1$

16 $3^{2x-2}\times2^{3x+2}\times2^{-3x}=2^{2x}$

, $\because 2^{x}=5$ $\therefore 2^{2x}=(2^{x})^2=(5)^2=25$

7

1 The left side $=\frac{2\times2^{1-x}\times3^{2x-1}\times2^{4x-2}}{2^{3x}\times3^{2x}}$

$=2^{1+1-x+4x-2-3x}\times3^{2x-1-2x}$

$=2^0\times3^{-1}=\frac{1}{3}=$ The right side.

2 The left side $=\frac{3^{3x-3}\times2^{3x}}{2^{2x}\times(\sqrt{2})^{2x}\times3^{2x}\times(\sqrt{3})^{2x}}$

$=\frac{3^{3x-3}\times2^{3x}}{2^{2x}\times2^{x}\times3^{2x}\times3^{x}}$

$=3^{3x-3-2x-x}\times2^{3x-2x-x}$

$=3^{-3}=\frac{1}{3^3}=\frac{1}{27}=$ The right side.

8

$\frac{2^{3x}\times3^{2x}}{2^{x}\times3^{2x}}=64$ $\therefore 2^{3x-x}=64$ $\therefore 2^{2x}=64$

$\therefore 4^{x}=64$ $\because 4^{-x}=\frac{1}{4^{x}}$ $\therefore 4^{-x}=\frac{1}{64}$

9

1 $a^4-b^4=(\sqrt{3})^4-(\sqrt{2})^4=3^2-2^2=9-4=5$

2 $\frac{a^4}{b^4}=\frac{(\sqrt{3})^4}{(\sqrt{2})^4}=\frac{3^2}{2^2}=\frac{9}{4}$

10

$(x^2-y^2)^3=\left((2\sqrt{2})^2-3^2\right)^3=\left(2^2\times(\sqrt{2})^2-3^2\right)^3$

$=(4\times2-9)^3=(8-9)^3=-1$

11

$2(ab)^2\left(\frac{a}{b}\right)^2=2\left(\frac{3}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{3}}\right)^2\times\left(\frac{3}{\sqrt{2}}\times\frac{\sqrt{3}}{\sqrt{2}}\right)^2$

$=2\left(\frac{3}{\sqrt{3}}\right)^2\times\left(\frac{3\sqrt{3}}{2}\right)^2$

$=2\times\frac{9}{3}\times\frac{27}{4}=\frac{81}{2}$

12

$$x^2 + (x\,zy)^2 = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{3}}\right)^2$$

$$= \frac{3}{4} + \left(\frac{\sqrt{2}}{4}\right)^2 = \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$$

13

$$\because \frac{a}{b} = \frac{3\sqrt{2}}{2} \times \frac{\sqrt{2}}{\sqrt{3}} = \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\therefore \left(\frac{a}{b}\right)^2 - 3\left(\frac{b}{a}\right)^2 = \left(\sqrt{3}\right)^2 - 3 \times \left(\frac{1}{\sqrt{3}}\right)^2 = 3 - 1 = 2$$

14

1 $3\,(x + y)^4\,(x - y)^4 = 3\left(\left(2 + \sqrt{3}\right)^4\left(2 - \sqrt{3}\right)^4\right)$

$= 3\left(\left(2 + \sqrt{3}\right)\left(2 - \sqrt{3}\right)\right)^4$

$= 3\,(4 - 3)^4 = 3$

2 $\left(\frac{x - y}{x + y}\right)^2 = \left(\frac{2 - \sqrt{3}}{2 + \sqrt{3}}\right)^2 = \left(\frac{2 - \sqrt{3}}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}\right)^2$

$= \left(\frac{4 - 4\sqrt{3} + 3}{4 - 3}\right)^2 = \left(7 - 4\sqrt{3}\right)^2$

$= 49 - 56\sqrt{3} + 48 = 97 - 56\sqrt{3}$

15

$7\,a^6 + (1 - b)^{-3} = 7 \times \left(\frac{1}{\sqrt{2}}\right)^6 + (1 + 1)^{-3}$

$= 7 \times \frac{1}{2^3} + 2^{-3} = 7 \times \frac{1}{2^3} + \frac{1}{2^3} = \frac{7}{8} + \frac{1}{8} = \frac{8}{8} = 1$

16

1 $x^{-2}\,y^{-4} = \frac{1}{x^2\,y^4} = \frac{1}{3^2 \times \left(\sqrt{2}\right)^4} = \frac{1}{9 \times 4} = \frac{1}{36}$

2 $(x^{-2} \times y^4)^{-2} = x^4\,y^{-8} = \frac{x^4}{y^8} = \frac{3^4}{\left(\sqrt{2}\right)^8} = \frac{81}{16}$

3 $\left(\frac{x}{y}\right)^{-3} = \left(\frac{y}{x}\right)^3 = \frac{y^3}{x^3} = \frac{\left(\sqrt{2}\right)^3}{3^3} = \frac{2\sqrt{2}}{27}$

17

1 d	2 c	3 d	4 c	5 c
6 a	7 b	8 c	9 b	10 c
11 c	12 a	13 c	14 a	15 c
16 d	17 c	18 b	19 b	20 a
21 d	22 a	23 c		

18

1 3	2 1	3 zero	4 zero	5 3
6 $\frac{1}{4}$	7 $\left(-\sqrt{2}\right)^{24}$	8 3	9 $\mathbb{R} - \{5\}$	10 1
11 1	12 $\frac{36}{25}$	13 125	14 35	15 $\frac{3}{7}$

19

The expression $= \frac{1}{2}\,(2\,x^4\,y^2)^2 \times (x\,y)^2$

$= \frac{1}{2} \times 4\,x^8\,y^4 \times x^2\,y^2$

$= 2 \times x^{8+2} \times y^{4+2}$

$= (2\,x^{10}\,y^6)\ \text{cm}^2$

20

The number of seconds $= \frac{2.7 \times 10^{11}}{5.4 \times 10^9}$

$= 0.5 \times 10^2 = 50$ seconds.

21

1 $\frac{1}{4}$	2 $\sqrt{2}$	3 16	4 2025

22

1 c	2 c	3 c	4 b	5 c

Answers of Exercise 11

1

1 $\because 5^n = 5^2$ $\therefore n = 2$

2 $\because 2^{-n} = 2^5$ $\therefore -n = 5$ $\therefore n = -5$

3 $\because 2^{n-1} = 2^5$ $\therefore n - 1 = 5$ $\therefore n = 6$

4 $\because 3^{n-2} = 3^4$ $\therefore n - 2 = 4$ $\therefore n = 6$

5 $\because 3^{n-2} = 3^{\text{zero}}$ $\therefore n - 2 = \text{zero}$ $\therefore n = 2$

6 $\because 2^{n-3} = 2^{-2}$ $\therefore n - 3 = -2$ $\therefore n = 1$

7 $\because 3^{n-2} = 3^{-2}$ $\therefore n - 2 = -2$ $\therefore n = \text{zero}$

8 $\because \left(\sqrt{3}\right)^{n-1} = \left(\sqrt{3}\right)^4$ $\therefore n - 1 = 4$ $\therefore n = 5$

9 $\because \left(\frac{2}{5}\right)^{2n-1} = \left(\frac{2}{5}\right)^3$ $\therefore 2n - 1 = 3$

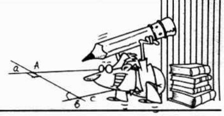
$\therefore 2n = 4$ $\therefore n = 2$

10 $\because \left(\frac{3}{5}\right)^{n+2} = \left(\frac{5}{3}\right)^3$ $\therefore \left(\frac{3}{5}\right)^{n+2} = \left(\frac{3}{5}\right)^{-3}$

$\therefore n + 2 = -3$ $\therefore n = -5$

11 $\because \left(\frac{2}{3}\right)^{n-4} = \frac{9}{4}$ $\therefore \left(\frac{2}{3}\right)^{n-4} = \left(\frac{3}{2}\right)^2$

$\therefore \left(\frac{2}{3}\right)^{n-4} = \left(\frac{2}{3}\right)^{-2}$ $\therefore n - 4 = -2$ $\therefore n = 2$



12 $\because \left(\frac{2}{3}\right)^{n+5} = \left(\frac{27}{8}\right)^{-2}$

$\therefore \left(\frac{2}{3}\right)^{n+5} = \left(\left(\frac{3}{2}\right)^{3}\right)^{-2} = \left(\frac{3}{2}\right)^{-6}$

$\therefore \left(\frac{2}{3}\right)^{n+5} = \left(\frac{2}{3}\right)^{6}$ $\therefore n + 5 = 6$ $\therefore n = 1$

13 $\because 5^{2n-4} = 7^{2n-4}$ $\therefore 2n - 4 = 0$

$\therefore 2n = 4$ $\therefore n = 2$

14 either $n = 3$ or $n - 4 = 0$ $\therefore n = 4$

15 $\because 3^{2} \times 3^{n-4} = 1$ $\therefore 3^{n-2} = 1$

$\therefore n - 2 = 0$ $\therefore n = 2$

16 $\because 2 \times 2^{2n+6} = \frac{1}{32}$ $\therefore 2^{2n+7} = 2^{-5}$

$\therefore 2n + 7 = -5$ $\therefore 2n = -12$ $\therefore n = -6$

17 $\because \left(\sqrt{2}\right)^{6} \times \left(\sqrt{2}\right)^{n-3} = 1$ $\therefore \left(\sqrt{2}\right)^{n+3} = 1$

$\therefore n + 3 = 0$ $\therefore n = -3$

18 $\because \frac{1}{\left(\sqrt{3}\right)^{6}} \times \left(\sqrt{3}\right)^{n+2} = 1$

$\therefore \left(\sqrt{3}\right)^{-6} \times \left(\sqrt{3}\right)^{n+2} = 1$ $\therefore \left(\sqrt{3}\right)^{n-4} = 1$

$\therefore n - 4 = 0$ $\therefore n = 4$

2

1 $\because x^{2} - 4 = 0$ $\therefore x^{2} = 4$

$\therefore x = \pm 2$ $\therefore$ **The S.S.** $= \{2 , -2\}$

2 $\because 2^{x^{2}-9} = 1$ $\therefore x^{2} - 9 = 0$

$\therefore x^{2} = 9$ $\therefore x = \pm 3$

$\therefore$ **The S.S.** $= \{-3 , 3\}$

3 $\because 2^{x^{2}-x} = 4$ $\therefore 2^{x^{2}-x} = 2^{2}$

$\therefore x^{2} - x = 2$ $\therefore x^{2} - x - 2 = 0$

$\therefore (x - 2)(x + 1) = 0$ $\therefore x - 2 = 0$

$\therefore x = 2$ or $x + 1 = 0$

$\therefore x = -1$ $\therefore$ **The S.S.** $= \{-1 , 2\}$

4 $\because 5^{|x|} = 5^{3}$ $\therefore |x| = 3$ $\therefore x = \pm 3$

$\therefore$ **The S.S.** $= \{3 , -3\}$

5 $\because 2^{5x-15} = 2^{6x+3}$ $\therefore 5x - 15 = 6x + 3$

$\therefore 5x - 6x = 15 + 3$ $\therefore -x = 18$

$\therefore x = -18$ $\therefore$ **The S.S.** $= \{-18\}$

6 $\because \left(\sqrt{3}\right)^{2x-6} = \left(\sqrt{3}\right)^{x+5}$ $\therefore 2x - 6 = x + 5$

$\therefore x = 11$ $\therefore$ **The S.S.** $= \{11\}$

7 $\because 3^{n-1} \times 2^{-(n-1)} = \left(\frac{3}{2}\right)^{3}$

$\therefore \frac{3^{n-1}}{2^{n-1}} = \left(\frac{3}{2}\right)^{3}$ $\therefore \left(\frac{3}{2}\right)^{n-1} = \left(\frac{3}{2}\right)^{3}$

$\therefore n - 1 = 3$ $\therefore n = 4$ $\therefore$ **The S.S.** $= \{4\}$

8 $\because \frac{3^{x-1}}{5^{x-1}} = \frac{9}{25}$ $\therefore \left(\frac{3}{5}\right)^{x-1} = \left(\frac{3}{5}\right)^{2}$

$\therefore x - 1 = 2$ $\therefore x = 3$ $\therefore$ The S.S. $= \{3\}$

3

1 $\because \frac{2^{n} \times 3^{2n+2}}{2^{n} \times 3^{2n}} = 3^{n}$ $\therefore 3^{2n+2-2n} = 3^{n}$

$\therefore 3^{2} = 3^{n}$ $\therefore n = 2$

2 $\because \frac{6^{2n-3}}{(2 \times 3)^{n-1}} = 6$ $\therefore \frac{6^{2n-3}}{6^{n-1}} = 6$

$\therefore 6^{2n-3-n+1} = 6$ $\therefore 6^{n-2} = 6^{1}$ $\therefore n = 3$

3 $\because \frac{3^{n-1} \times 2^{2n-2}}{2^{n-1} \times 3^{n-1}} = 1$ $\therefore 2^{2n-2-n+1} = 2^{0}$

$\therefore n - 1 = 0$ $\therefore n = 1$

4 $\because \frac{3^{n} \times 2^{3n}}{3^{n+1} \times 2^{2n+2}} = \frac{1}{3}$

$\therefore 3^{n-n-1} \times 2^{3n-2n-2} = 3^{-1}$

$\therefore 3^{-1} \times 2^{n-2} = 3^{-1}$ $\therefore 2^{n-2} = 1$ $\therefore 2^{n-2} = 2^{0}$

$\therefore n - 2 = 0$ $\therefore n = 2$

5 $\because \frac{2^{4n} \times 3^{2n}}{3^{2n} \times 2^{2n}} = \frac{1}{2^{4}}$ $\therefore 2^{2n} = 2^{-4}$ $\therefore 2n = -4$

$\therefore n = -2$

6 $\because \frac{2^{2n-2} \times 2^{n+3}}{2^{3n}} = 2n^{2}$ $\therefore 2^{2n-2+n+3-3n} = 2n^{2}$

$\therefore 2 = 2n^{2}$ $\therefore n^{2} = 1$ $\therefore n = \pm 1$

7 $\because \frac{2^{2n} \times 7^{2n} \times 2^{2n+2}}{2^{2} \times 7^{n} \times 2^{4n}} = 7^{2}$

$\therefore 2^{2n+2n+2-2-4n} \times 7^{2n-n} = 7^{2}$

$\therefore 2^{0} \times 7^{n} = 7^{2}$ $\therefore 7^{n} = 7^{2}$ $\therefore n = 2$

4

1 $\because (x - 4)^{5} = 2^{5}$ $\therefore x - 4 = 2$ $\therefore x = 6$

$\therefore$ **The S.S.** $= \{6\}$

2 $\because \frac{1}{(x + 9)^{4}} = \left(\frac{1}{10}\right)^{4}$ $\therefore (x + 9)^{-4} = (10)^{-4}$

$\therefore x + 9 = \pm 10$ $\therefore x = 1$ or $x = -19$

$\therefore$ **The S.S.** $= \{1 , -19\}$

3 $\because (x^{2} - x)^{5} = 2^{5}$ $\therefore x^{2} - x = 2$

$\therefore x^{2} - x - 2 = 0$ $\therefore (x + 1)(x - 2) = 0$

$\therefore$ either $x + 1 = 0$ $\therefore x = -1$ or $x - 2 = 0$

$\therefore x = 2$ $\therefore$ **The S.S.** $= \{-1 , 2\}$

4 $\because x^2 - x = 0$ $\therefore x(x-1) = 0$

$\therefore x = 0$ or $x - 1 = 0$ $\therefore x = 1$

$\therefore$ **The S.S. =** $\{0 , 1\}$

5 $\because 5^{x^2-5x} = \frac{16}{10000} = \left(\frac{2}{10}\right)^4$

$\therefore 5^{x^2-5x} = \left(\frac{10}{2}\right)^{-4} = 5^{-4}$

$\therefore x^2 - 5x = -4$ $\therefore x^2 - 5x + 4 = 0$

$\therefore (x-4)(x-1) = 0$ $\therefore x - 4 = 0$

$\therefore x = 4$ or $x - 1 = 0$

$\therefore x = 1$ $\therefore$ **The S.S. =** $\{1 , 4\}$

6 $\because 5^{x^2} = 5^{2x+8}$ $\therefore x^2 = 2x + 8$

$\therefore x^2 - 2x - 8 = 0$

$\therefore (x-4)(x+2) = 0$ either $x - 4 = 0$ then $x = 4$

or $x + 2 = 0$ then $x = -2$ $\therefore$ **The S.S. =** $\{4 , -2\}$

7 $\because 3^{2x^2-2} = \frac{1}{3^{3x}}$ $\therefore 3^{2x^2-2} = 3^{-3x}$

$\therefore 2x^2 - 2 = -3x$ $\therefore 2x^2 + 3x - 2 = 0$

$\therefore (2x-1)(x+2) = 0$ either $2x - 1 = 0$

$\therefore x = \frac{1}{2}$ or $x + 2 = 0$ $\therefore x = -2$

$\therefore$ **The S.S. =** $\left\{-2 , \frac{1}{2}\right\}$

5

$\because \frac{2^{2n} \times 3^{2n} \times 2^{2n}}{2^{4n} \times 3^{2n+4}} = 3^{-2x}$

$\therefore 2^{2n+2n-4n} \times 3^{2n+2n-4} = 3^{-2x}$

$\therefore 2^0 \times 3^{-4} = 3^{-2x}$ $\therefore 3^{-4} = 3^{-2x}$

$\therefore -2x = -4$ $\therefore x = 2$

6

$\because \frac{3^{4x} \times \cancel{2^{2x}}}{\cancel{2^{2x}} \times 3^{2x} \times 3^{2x}} = 3^{4x-2x-2x} = 1$

$\therefore 3^{y-1} = 1$ $\therefore y - 1 = 0$ $\therefore y = 1$

7

$\because \frac{3^{3x-3} \times 2^{3x}}{2^{2x} \times (\sqrt{2})^{2x} \times 3^{2x} \times (\sqrt{3})^{2x}} = 3^x$

$\therefore \frac{3^{3x-3} \times 2^{3x}}{2^{2x} \times 2^x \times 3^{2x} \times 3^x} = 3^x$

$\therefore \frac{3^{3x-3} \times \cancel{2^{3x}}}{\cancel{2^{3x}} \times 3^{3x}} = 3^x$ $\therefore 3^{3x-3-3x} = 3^x$

$\therefore 3^{-3} = 3^x$ $\therefore x = -3$

8

$\frac{7^x \times 2^x \times 3^x}{2^x \times 7^x} = 3^{2-y}$ $\therefore 3^x = 3^{2-y}$

$\therefore x = 2 - y$ $\therefore x + y = 2$

9

$\because \left(\sqrt{\frac{2}{3}}\right)^{-x} = \left(\sqrt{\frac{2}{3}}\right)^4$ $\therefore -x = 4$ $\therefore x = -4$

$\therefore \left(\frac{3}{2}\right)^{x+1} = \left(\frac{3}{2}\right)^{-4+1} = \left(\frac{3}{2}\right)^{-3} = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

10

$\because \frac{7^{2n} \times 5^{4n} \times 3^{4n}}{7^{-n} \times 3^{4n} \times 5^{4n}} = 7^3$

$\therefore 7^{2n+n} \times 5^{4n-4n} \times 3^{4n-4n} = 7^3$ $\therefore 7^{3n} = 7^3$

$\therefore 3n = 3$ $\therefore n = 1$ $\therefore 6^{2n} = 6^2 = 36$

11 $\because 3^x = 3^3$ $\therefore x = 3$ $\therefore 4^{3+y} = 1$

$\therefore 3 + y = 0$ $\therefore y = -3$

12

1 c	**2** d	**3** a	**4** b	**5** d
6 b	**7** b	**8** c	**9** b	**10** b
11 a	**12** a	**13** b	**14** a	**15** b

13

1 zero or 1	**2** – 5	**3** 2	**4** 7
5 1	**6** – 1	**7** 3	**8** 2
9 – 1	**10** 1	**11** – 1	**12** 2
13 4 , 5			

14

$\because$ The number of the small squares in the chess board of side length 4 cm. = 64 squares.

$\therefore$ The area of one small square = $4 \times 4 = 16$ cm.^{2}

$\therefore$ The area of the chess board = $16 \times 64 = 2^n$

$\therefore 2^4 \times 2^6 = 2^n$ $\therefore 2^{10} = 2^n$ $\therefore n = 10$

15

1 $\because x^{x+2} = 4^{x+2}$ $\therefore x = \pm 4$

(because : $x + 2 = 4 + 2 = 6$ "even number"

or : $x + 2 = -4 + 2 = -2$ "even number")

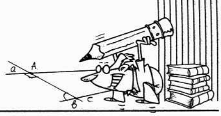
or $x + 2 = 0$ $\therefore x = -2$ $\therefore x = \pm 4$ or -2

2 $\because a^{x+3} - 1 = (a^2 - 1)(a^2 + 1)(a^4 + 1)$

$a^{x+3} - 1 = (a^4 - 1)(a^4 + 1)$

$\therefore a^{x+3} - 1 = a^8 - 1$ $\therefore a^{x+3} = a^8$

$\therefore x + 3 = 8$ $\therefore x = 5$



Answers of Exercise 12

1

1 2 2 18 3 $-\frac{1}{2}$ 4 $-\frac{2}{9}$

2

1 $(\sqrt{5})^5 \div (\sqrt{5})^3 + 2\sqrt{3} \times \sqrt{3}$

$= (\sqrt{5})^{5-3} + 2 \times (\sqrt{3})^{1+1} = (\sqrt{5})^2 + 2(\sqrt{3})^2$

$= 5 + 6 = 11$

2 $2^3 \times (\sqrt{3})^3 \times \sqrt{3} - (\sqrt{2})^7 \div (\sqrt{2})^5$

$= 2^3 \times (\sqrt{3})^{3+1} - (\sqrt{2})^{7-5}$

$= 2^3 \times (\sqrt{3})^4 - (\sqrt{2})^2 = 8 \times 9 - 2 = 70$

3 $(\sqrt{3})^{-3} \times (\sqrt{3})^3 + (\sqrt{3})^{-4} \times (\sqrt{3})^{10}$

$= (\sqrt{3})^{-3+3} + (\sqrt{3})^{-4+10}$

$= 1 + (\sqrt{3})^6 = 1 + 27 = 28$

4 $2^4 \times (\sqrt{5})^4 - (\sqrt{5})^{-3} \times (\sqrt{5})^6 \div (\sqrt{5})^3$

$= 2^4 \times (\sqrt{5})^4 - (\sqrt{5})^{-3+6-3} = 16 \times 25 - 1 = 399$

3

1 $\dfrac{(\sqrt{3})^7 \times (\sqrt{3})^{-} - (\sqrt{3})^2}{(\sqrt{3})^7 \times (\sqrt{3})^{-5} + (\sqrt{3})^2} = \dfrac{(\sqrt{3})^2 - (\sqrt{3})^2}{(\sqrt{3})^2 + (\sqrt{3})^2}$

$= \dfrac{0}{3+3} = \text{zero}$

2 $\dfrac{2(\sqrt{3})^5 \div (\sqrt{3})^3}{2\sqrt{3} + 3 - 2\sqrt{3} + 1} = \dfrac{2(\sqrt{3})^2}{4} = \dfrac{3}{2}$

3 $\dfrac{2^3 \times (\sqrt{2})^3 \times 3 \times \sqrt{2}}{6 + 2\sqrt{12} + 2 - 2\sqrt{12}} = \dfrac{8 \times 3 \times (\sqrt{2})^4}{8} = 3 \times 4 = 12$

4

1 $\dfrac{b^4 - a^4}{b^2 + a^2} = \dfrac{(b^2 - a^2)(b^2 + a^2)}{(b^2 + a^2)} = b^2 - a^2$

$= (\sqrt{3})^2 - (\sqrt{2})^2 = 3 - 2 = 1$

2 $\dfrac{a^3 + b^3}{a + b} = \dfrac{(a + b)(a^2 - ab + b^2)}{(a + b)}$

$= a^2 - ab + b^2$

$= (\sqrt{2})^2 - \sqrt{2} \times \sqrt{3} + (\sqrt{3})^2$

$= 2 - \sqrt{6} + 3 = 5 - \sqrt{6}$

5

1 a 2 d 3 a 4 a 5 d

6

1 $\because$ The total area of the cube $= 6\ell^2$

$\therefore 6\ell^2 = 3.375 \times 10^2 \quad \therefore \ell^2 = \dfrac{3.375 \times 10^2}{6}$

$\therefore \ell = \sqrt{\dfrac{3.375 \times 10^2}{6}}$

Using the calculator $\ell = 7.5$ cm.

2 $\because$ The volume of the cube $= \ell^3$

$\therefore$ The volume of the cube $= (7.5)^3$

Using the calculator ,

the volume of the cube $= 421.875$ cm^3.

7

$\because$ The volume of the sphere $= \frac{4}{3}\pi r^3$

$\therefore 3.8808 \times 10^4 = \frac{4}{3}\pi r^3$

$\therefore r^3 = \dfrac{3.8808 \times 10^4 \times 3}{4\pi} \quad \therefore r = \sqrt[3]{\dfrac{3.8808 \times 10^4 \times 3}{4 \times \frac{22}{7}}}$

Using the calculator r = 21 cm.

8

$\because$ The volume of the cone $= \frac{1}{3}\pi r^2 h$

$\therefore 7.7 \times 10^2 = \frac{1}{3}\pi \times 7^2 \times h$

$\therefore 7.7 \times 10^2 = \frac{49}{3} \times \pi \times h$

$\therefore h = \dfrac{7.7 \times 10^2 \times 3}{49 \times \frac{22}{7}}$ Using the calculator h = 15 cm.

9

$c = 2.5 \times 10^4 (1 + 9.8 \times 10^{-2})^{12}$

Using the calculator we get:

$c = 76765.85477 \simeq 76766$ to the nearest pound.

10

1 $y = 11.7\,(1.02)^6$ Using the calculator

$y = 13.17610031 \simeq 13$ million persons

2 $y = 11.7\,(1.02)^{-5}$ Using the calculator

$y = 10.59\,705\,048 \simeq 11$ million persons

11

$\dfrac{x^7 y^8 - y}{(x + y)^9} = \dfrac{y(x^7 y^7 - 1)}{(x + y)^9}$

$, x \times y = (2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 3 = 1$

$, x + y = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

$\therefore \dfrac{x^7 y^8 - y}{(x + y)^9} = \dfrac{(2 - \sqrt{3})(1^7 - 1)}{4^9} = \text{zero}$

12

$\because \ell + m = \frac{1}{2}(3^x + 3^{-x}) + \frac{1}{2}(3^x - 3^{-x})$

$= \frac{1}{2}(3^x + 3^{-x} + 3^x - 3^{-x})$

$= \frac{1}{2} \times (2 \times 3^x) = 3^x$

$, \ell - m = \frac{1}{2}(3^x + 3^{-x}) - \frac{1}{2}(3^x - 3^{-x})$

$= \frac{1}{2}(3^x + 3^{-x} - 3^x + 3^{-x})$

$= \frac{1}{2} \times (2 \times 3^{-x}) = 3^{-x}$

$\therefore \ell^2 - m^2 = (\ell - m)(\ell + m) = 3^x \times 3^{-x} = 3^{zero} = 1$

Answers of the general exercises on unit two

First : Completion questions

1 $\frac{\sqrt{2}}{4}$ **2** 5 **3** $\frac{9}{4}$ **4** 2

5 4 **6** 8 **7** $\left(-\sqrt{11}\right)^{24}$ **8** zero

9 1 **10** 20 **11** 1 **12** – 1

13 1 , x^2 **14** 1 **15** 2 **16** 2

17 $-\frac{1}{2}$ **18** 128 **19** – 1 **20** 1

Second : Multiple choice questions

1 c **2** b **3** c **4** c **5** c **6** a

7 b **8** d **9** a **10** d **11** c **12** b

13 c **14** a **15** a **16** a **17** b **18** c

19 d **20** a

Third : Essay questions

1 **1** $\frac{1}{3}$ **2** 4 **3** $\frac{8}{27}$ **4** 25 **5** $\frac{1}{3}$

6 $\frac{1}{7}$ **7** $\frac{1}{8}$ **8** 10000 **9** 4

2 **1** $\frac{1}{3}$ **2** $-\frac{1}{4}$ **3** $2\sqrt{2}$ **4** $9\sqrt{3}$ **5** $\frac{4}{9}$

6 3

3 **1** 8 **2** 25 **3** 36 **4** 72

5 $\frac{64}{729}$ **6** $81\sqrt{3}$

4 **1** $\sqrt{3}$ **2** 1 **3** $\frac{1}{108}$ **4** $3\sqrt{2}$

5 **1** $\frac{1}{36}$ **2** $\frac{81}{16}$ **3** $\frac{2\sqrt{2}}{27}$

6 $\frac{7}{8}$

7 **1** 1 **2** $97 - 56\sqrt{3}$

8 1

9 **1** 5 **2** $\frac{9}{4}$

10 – 1

11 $\frac{27}{8}$

12 , **13** Prove by yourself.

14 **1** 5 **2** 3 **3** 6 **4** 5

5 zero **6** 2

15 Prove by yourself.

16 $\frac{1}{64}$

17 $2^2 \times 3^{4-4x}$, 4

18 **1** 7.5 length units **2** 421.875 cube units.

19 21 cm.

Answers of worksheets in algebra and statistics

Worksheet 1

1

(1) (c) (2) (d) (3) (c)

(4) (c) (5) (a) (6) (c)

2

(1) $(a+1)(a-4)$ (2) ± 2 (3) -3

(4) $(x-1)$ (5) $(x-1)$ (6) $6x, x, 4$

3

(1) $(a^2+5)(x-y)$ (2) $3(x-2)(x-3)$

4

(1) 1 (2) 12 (3) 2

Try by yourself to get other values of k

Worksheet 2

1

(1) $(2x-3)$ (2) 5 (3) 1 , 2

(4) $2x^2, 7x, 5$ (5) 21 (6) $-5a$

2

(1) (c) (2) (a) (3) (b)

(4) (b) (5) (b) (6) (b)

3

The dimensions are $(2x+5)$, $(x+7)$ cm.

The perimeter = 30 cm.

4

[a] (1) $(2x+3)(x-2)$ (2) $(2x+1)(x-4)$

[b] $c = 2$, $(2x-5)(x+3)$

Try to get other values for c by yourself.

Worksheet 3

1

(1) (c) (2) (d) (3) (a)

(4) (b) (5) (a) (6) (d)

2

(1) 5 (2) 9 (3) $3x, 2$

(4) $6xy, 9y^2, x$ (5) $3, 25a^2, 9$ (6) 30

3

[a] 100

[b] The side length of the square = $(10x+4)$ cm.

The perimeter of the square = 56 cm.

4

(1) $(x-5)(x-2)$ (2) $(2x-3)(x+2)$

(3) $(5x+3)^2$ (4) $2x(x-5)^2$

Worksheet 4

1

(1) (a) (2) (d) (3) (d)

(4) (b) (5) (b) (6) (c)

2

(1) 5 (2) 38 (3) -5

(4) $(x-2)(x+2)$ (5) 50 (6) $1, x$

3

(1) $(2x-3)(x+2)$ (2) $3(x-3)(x+3)$

(3) $(0.8x+0.3y)(0.8x-0.3y)$

4

[a] 9800 **[b]** $(3b-4)(3b+4)(a-5b)$

Worksheet 5

1

(1) (b) (2) (d) (3) (b)

(4) (a) (5) (d) (6) (c)

2

(1) $x-5$ (2) 4 (3) 16

(4) 8 (5) $(x-3)$ (6) ± 6

3

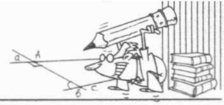
(1) $(x-2)(x+8)$ (2) $(x-3)(x^2+3x+9)$

(3) $(2a-3b)(a-2b)$

4

[a] 10000

[b] $(x-2)(x^2+2x+4)(x+2)(x^2-2x+4)$



Worksheet 6

1

(1) (c) (2) (b) (3) (d)

(4) (b) (5) (b) (6) (a)

2

(1) $(x - y - 1)$ (2) -16

(3) $(3x + 2y)(9x^2 - 6xy + 4y^2)$ (4) 16

(5) $(3x - 7)(x - 1)$ (6) $(x + a)(x - b)$

3

(1) $\frac{1}{2}(x - 2)(x^2 + 2x + 4)$

(2) $(2x + y + 3)(2x + y - 3)$

(3) $(x + y)(x^2 - xy + y^2 - 1)$

4

[a] 63.5

[b] $(x - y)(x^2 + xy + y^2)(x + y)(x^2 - xy + y^2)$

Worksheet 7

1

(1) (b) (2) (d) (3) (a)

(4) (c) (5) (a) (6) (b)

2

(1) 125 (2) $(a + 2)$ (3) 1

(4) $(5x - 7)$ (5) 40 (6) $4x^2$

3

(1) $3x(x + 5)(x - 5)$ (2) $x(x - 1)(x^2 + x + 1)$

(3) $(2x - 9)(3x + 2)$ (4) $(x - 6)(y - 5)$

4

(1) $(2x - 5y)(x + y)(2x + 5y)(x - y)$

(2) $(x - y)(x + 2y)(x + y)(x - 2y)$

Worksheet 8

1

(1) (b) (2) (a) (3) (d)

(4) (c) (5) (c) (6) (b)

2

(1) $4x$, $14xy$ (2) 15 (3) -81

(4) $\varnothing$ (5) 9 (6) $\{0 , 4\}$

3

(1) $(5x + 6)(x - 2)$ (2) $(a + 1)(a + 3)(a - 3)$

(3) $(x^2 + 2y^2 - 2xy)(x^2 + 2y^2 + 2xy)$

4

(1) S.S. = $\{-3 , 10\}$ (2) S.S. = $\{-1 , 3\}$

Worksheet 9

1

(1) (c) (2) (c) (3) (a)

(4) (a) (5) (b) (6) (c)

2

(1) $(2x - 1)$ (2) 6 (3) $2x^2$

(4) ± 1 (5) 28 (6) 6 , 1

3

[a] (1) $(2x + 3)(2x - 5)$

(2) $(x^2 - 5y^2 - 3xy)(x^2 - 5y^2 + 3xy)$

[b] S.S. = $\{-\frac{2}{3} , 3\}$

4

[a] 1 , $-\frac{1}{2}$ [b] 25 cm.

Worksheet 10

1

(1) (a) (2) (a) (3) (c)

(4) (c) (5) (d) (6) (c)

2

(1) $\{0 , 2\}$ (2) 1 (3) 4

(4) $(x + 1)$ (5) $\mathbb{R} - \{5\}$ (6) 10

3

[a] Prove by yourself.

[b] (1) $(a + b)(3x + 2y)$

(2) $(2x - 5)(4x^2 + 10x + 25)$

4

[a] The dimensions of the rectangle are 7 m. and 12 m.

The perimeter of the rectangle = 38 m.

[b] $\frac{1}{36}$

Worksheet 11

1

(1) (d) (2) (d) (3) (d)

(4) (d) (5) (b) (6) (c)

2

(1) $\frac{9}{4}$ (2) 2 (3) 35

(4) 20 (5) $\{0 , 1\}$ (6) 2

3

[a] (1) $(2x - 3)(x + 2)$

(2) $(2x - 3)(4x^2 + 6x + 9)$

(3) $(x - 1)(x + y)$

[b] $n = 2$

4

[a] $(x^2 + y^2 - 3xy)(x^2 + y^2 + 3xy)$

[b] $\frac{27}{8}$

Worksheet 12

1

(1) (b) (2) (a) (3) (d)

(4) (a) (5) (a) (6) (d)

2

(1) 2 (2) $x + y$ (3) $\{-1 , 1\}$

(4) $(x + 3)$ (5) $a - b + 1$ (6) $(x + 5)$

3

(1) $2(x + 5y)(x - 5y)$ (2) $(5x + 7y)(x - 3y)$

(3) $x^3(2 - 7x)(4 + 14x + 49x^2)$

(4) $(x^2 + 2\ell^2 - 2x\ell)(x^2 + 2\ell^2 + 2x\ell)$

4

[a] $\frac{7}{8}$ [b] $\frac{1}{64}$

Worksheet 13

1

(1) (d) (2) (c) (3) (c)

(4) (b) (5) (c) (6) (d)

2

(1) zero , 1 (2) $1 + x^2$ (3) $\frac{3}{20}$

(4) 15 (5) $2a , -22ab$ (6) 1

3

[a] (1) $\frac{1}{2}(x - 2y)(x + 2y)$

(2) $(3x + 2y)(9x^2 - 6xy + 4y^2)$

(3) $(3a + b)(2a - 7b)$

(4) $(x^2 + 8 - 4x)(x^2 + 8 + 4x)$

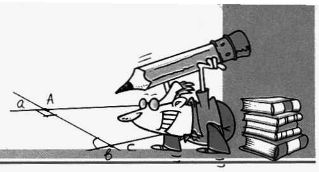
[b] Prove by yourself.

4

[a] Hatem's age = 5 years.

Hanan's age = one year.

[b] (1) $\frac{2}{5}$ (2) $\frac{8}{25}$



From the school book

Exercise 5 On areas of some geometric figures

1 Complete the following :

1. The area of the rhombus = the side length × .... = $\frac{1}{2}$ of the product of ....
2. The area of the square = the square of the length of .... = $\frac{1}{2}$ ....
3. The length of the middle base of the trapezium equals ....
4. The area of the trapezium = half of the sum of lengths of the two parallel bases × ....
   = the length of .... × its height
5. The base angles of the isosceles trapezium are ....
6. 📖 The diagonals of an isosceles trapezium are ....

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2 Find the area of the following figures :

1. A rhombus of side length 6 cm. and its height = 5 cm. « 30 cm^2 »
2. 📖 A rhombus whose side length 12 cm. and its height = 8 cm. « 96 cm^2 »
3. 📖 A rhombus whose diagonals lengths are 8 cm. and 10 cm. « 40 cm^2 »
4. A rhombus whose diagonal lengths are 24 cm. and 10 cm. « 120 cm^2 »
5. A square whose diagonal length = 10 cm. « 50 cm^2 »
6. 📖 A square whose diagonal length = 8 cm. « 32 cm^2 »
7. A trapezium whose bases lengths are 6 cm. and 8 cm. and its height = 12 cm. « 84 cm^2 »
8. A trapezium whose bases lengths are 8 cm. and 10 cm. and its height = 5 cm. « 45 cm^2 »
9. A trapezium whose middle base length is 7 cm. and its height = 6 cm. « 42 cm^2 »
10. A trapezium whose middle base length is 12 cm. and its height = 8 cm. « 96 cm^2 »

---

3 Choose the correct answer from those given :

1. The area of rhombus is 20 cm^2, the length of one of its diagonals is 5 cm., then the length of the other diagonal = ...
   (a) 8 cm. (b) 4 cm. (c) 10 cm. (d) 15 cm.
2. If the area of a square is 50 cm^2, then the length of its diagonal = ...
   (a) 25 cm. (b) 5 cm. (c) 10 cm. (d) 20 cm.
3. The area of the square whose side length is 6 cm. .... the area of the square whose diagonal length is 8 cm.
   (a) > (b) < (c) = (d) ≡
4. If the perimeter of a rhombus is 24 cm. and its area = 30 cm^2 then its height = ...
   (a) 4 cm. (b) 5 cm. (c) 6 cm. (d) 12 cm.

5 If the product of the lengths of the diagonals of a rhombus = 96 cm² and its height is 6 cm., then its side length = ...

(a) 12 cm. (b) 8 cm. (c) 6 cm. (d) 4 cm.

6 The trapezium in which the lengths of its two parallel bases are 15 cm. and 11 cm. Its middle base is with length ....

(a) 26 cm. (b) 15 cm. (c) 13 cm. (d) 11 cm.

7 If the area of a trapezium is 32 cm² and its height is 4 cm., then the length of its middle base = ...

(a) 4 cm. (b) 8 cm. (c) 14 cm. (d) 16 cm.

8 If the area of the trapezium is 450 cm², and the lengths of its two parallel bases are 24 cm. and 12 cm. , then its height = ...

(a) 12.5 cm. (b) 25 cm. (c) 36 cm. (d) 52 cm.

9 The trapezium in which the length of one of its parallel bases is 15 cm., and its area is 108 cm² and its height is 8 cm., then the length of the other base is ....

(a) 15 cm. (b) 4 cm. (c) 12 cm. (d) 27 cm.

10 The trapezium whose middle base length is x cm. and its height = $\frac{1}{2}$ the length of the middle base, its area = ... cm².

(a) x^2 (b) $\frac{x^2}{2}$ (c) $\frac{x^2}{4}$ (d) $\frac{x^2}{8}$

---

4 A square whose area equals the area of the rectangle whose dimensions are 2 cm. and 9 cm. Find the length of its diagonal. « 6 cm. »

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5 Two land pieces are equal in area , the first is in the shape of a square and the second is in the shape of a rhombus whose diagonals lengths are 8 metres and 16 metres. Find the perimeter of the square-shaped piece. « 32 cm. »

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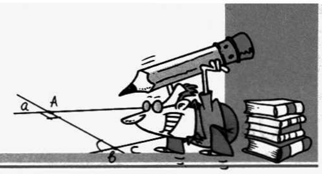
6 📖 Two pieces of land have equal areas , one of them has the shape of a rhombus whose diagonals are 18 m. and 24 m. , and the other one has the shape of a trapezium whose height is 12 m. Find the length of its middle base. « 18 m. »

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7 A rhombus whose diagonals are of lengths 12 cm. and 16 cm. Find its height. « 9.6 cm. »

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8 📖 Find the area of the rhombus whose perimeter is 52 cm. and the length of one of its diagonals is 10 cm. « 120 cm². »

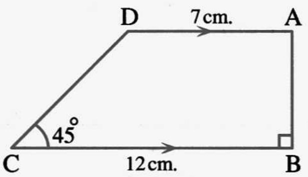


9 The perimeter of a rhombus is 64 cm. and the measure of one of its angles is 60°. Find its area. « $128\sqrt{3}$ cm.^{2} »

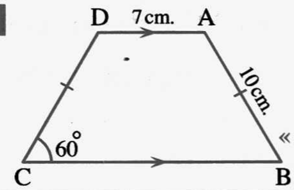
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10 **Find the area of each of the following figures by using the given data :**

1



2



« 47.5 cm.^{2} , $60\sqrt{3}$ cm.^{2} »

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11 If the ratio between the two lengths of the diagonals of a rhombus is 3 : 4 and the length of the smallest diagonal is 9 cm. Find the area of the rhombus. « 54 cm.^{2} »

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12 **A rhombus :** The ratio between the lengths of the two diagonals is 5 : 8 if its area = 2000 cm.^{2} Find the length of each of its diagonals. « 50 cm. , 80 cm. »

---

13 The length of the middle base of a trapezium is 30 cm. and the ratio between the lengths of its two parallel bases is 2 : 3 Find the length of each of them. and if its height = 24 cm. , find its area. « 24 cm. , 36 cm. , 720 cm.^{2} »

---

14 The area of a trapezium is 180 cm.^{2} and its height is 12 cm. Find the lengths of its parallel bases if the ratio between their lengths is 3 : 2 « 18 cm. , 12 cm. »

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15 A piece of land has the shape of a trapezium whose area is 4000 m.^{2}
The lengths of the two parallel bases and its height are of ratio 3 : 2 : 4 , respectively. Find the length of its middle base. « 50 cm. »

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16 Two pieces of land , the first is in the shape of a trapezium in which the lengths of its two parallel bases are 76 metres and 64 metres and the perpendicular distance between them is 45 metres and the second is in the shape of a rhombus whose diagonals lengths are 74 metres and 90 metres. The two pieces are exchanged by a rectangular piece of land whose area equals to the sum of areas of the trapezium and rhombus pieces. The ratio between its length and its width is 5 : 4 Find its dimensions. « 90 m. , 72 m. »

---

17 ABCD is a trapezium in which $\overline{AD} \parallel \overline{BC}$, X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{DC}$ If XY = 7 cm. , BC = 10 cm. and the area of the trapezium = 35 cm.^{2} , find the length of $\overline{AD}$ and the perpendicular distance between $\overline{AD}$ and $\overline{BC}$ « 4 cm. , 5 cm. »

18 ABCD is a trapezium in which $\overline{AD} \mathbin{/\!/} \overline{BC}$, AD = 27 cm. and BC = 45 cm.

If the area of Δ ABC = 225 cm^2, find the area of the trapezium. « 360 cm^2 »

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19 ABCD is a trapezium in which $\overline{AD} \mathbin{/\!/} \overline{BC}$, m ($\angle$ A) = 90°, BC = 4 cm., AD = 24 cm., BD = 30 cm. and $\overline{AF} \perp \overline{BD}$ to cut it at F where AF = 14.4 cm.

Find the area of the trapezium ABCD « 252 cm^2 »

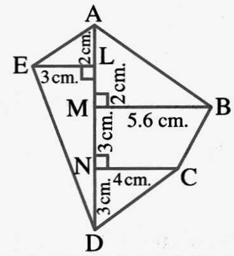
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20 **In the opposite figure :**

Each of $\overline{BM}$, $\overline{CN}$ and $\overline{EL}$ is perpendicular to $\overline{AD}$

Using the shown lengths on the figure,

find the area of the figure ABCDE « 46.6 cm^2 »



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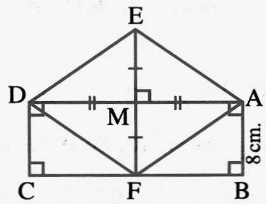
21 **In the opposite figure :**

ABCD is a rectangle of area 144 cm^2.

If AB = 8 cm., $\overline{AD} \perp \overline{EF}$

and M is the midpoint of each of $\overline{AD}$ and $\overline{EF}$,

find the area of the figure AFDE « 144 cm^2 »



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22 ABCD is a rectangle with AB = 6 cm., BC = 8 cm., X, Y, L and M are the midpoints of the sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively.

1. **Prove that :** The figure XYLM is a rhombus and find its area.
2. Find the height of the rhombus XYLM « 24 cm^2, 4.8 cm. »

For excellent pupils

23 The area of an isosceles trapezium is 120 cm.^{2}, its perimeter is 60 cm. and the length of its middle base is 20 cm. Find the lengths of its bases. « 12 cm., 28 cm. »

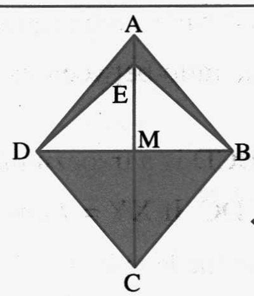
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24 **In the opposite figure :**

ABCD is a rhombus, its diagonals intersect at M,

AC + BD = 33 cm., BD : AC = 5 : 6 and E $\in \overline{AM}$ such that

ME = $\frac{2}{3}$ MA. Find the area of the shaded part. « 90 cm^2 »



From the school book

Exercise 6 On similarity

1 Complete each of the following statements :

1 If two polygons are similar , then the corresponding .... are equal in measure.

2 If two polygons are similar , then the corresponding .... are proportional.

3 If each of two polygons is similar to a third , then they are ....

4 The two triangles are similar if the corresponding .... are proportional.

5 If the measures of the corresponding angles in the two triangles are equal , then the two triangles are ....

6 If we have two polygons , their corresponding angles are .... and their corresponding sides lengths are .... , then the two polygons are similar.

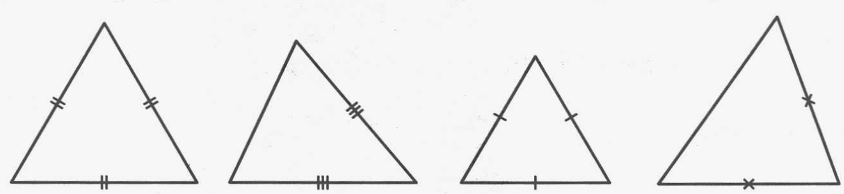
7 If the ratio between the lengths of two corresponding sides in two similar triangles is equal to 1 , then the two triangles are ....

8 If two polygons are similar and the ratio between the lengths of two corresponding sides is 3 : 4 , then the ratio between their perimeters is ....

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2 Choose the correct answer from those given ones :

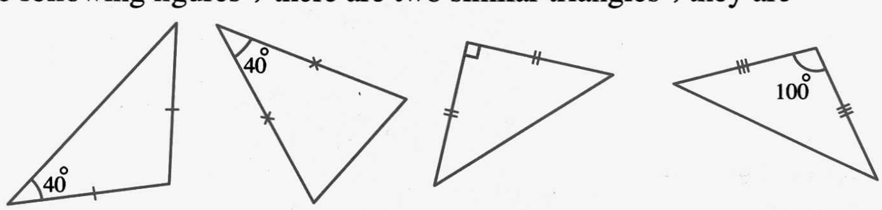
1 In the following figures , there are two similar triangles , they are ....



(1) (2) (3) (4)

(a) 1 , 2 (b) 1 , 3 (c) 1 , 4 (d) 2 , 4

2 In the following figures , there are two similar triangles , they are ....



(1) (2) (3) (4)

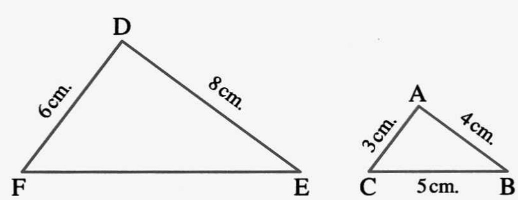
(a) 1 , 2 (b) 1 , 3 (c) 2 , 4 (d) 1 , 4

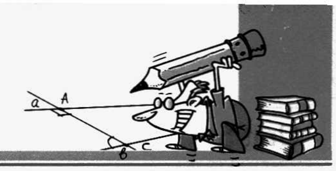
3 **In the opposite figure :**

If $\Delta ABC \sim \Delta DEF$, then EF = ...

(a) 5 cm. (b) 6 cm.

(c) 8 cm. (d) 10 cm.



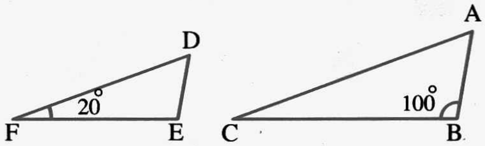


4 **In the opposite figure :**

If Δ ABC ~ Δ DEF , then m (∠ A) = ...

(a) 20° (b) 60°

(c) 80° (d) 100°



5 **In the opposite figure :**

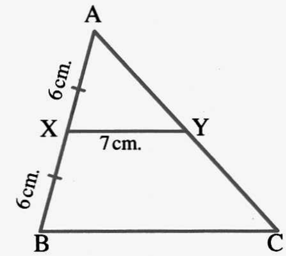
If Δ ABC ~ Δ AXY ,

AX = XB = 6 cm.

XY = 7 cm. , then BC = ...

(a) 6 cm. (b) 7 cm.

(c) 12 cm. (d) 14 cm.



6 If the ratio between the lengths of two corresponding sides of two squares is 1 and the perimeter of one of them is 20 cm. , then the area of the other square = ...

(a) 20 cm^2 (b) 25 cm^2 (c) 16 cm^2 (d) 25 cm.

7 If Δ ABC ~ Δ DEF and AB = $\frac{1}{5}$ DE , then perimeter of Δ ABC = ... perimeter of Δ DEF

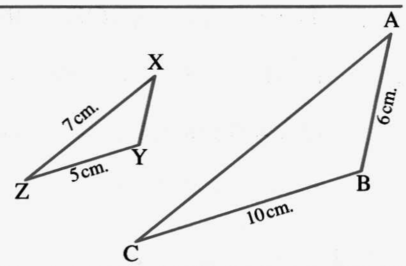
(a) 5 (b) 1 (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

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3 **In the opposite figure :**

Δ ABC ~ Δ XYZ

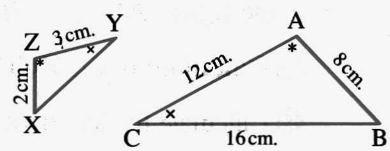
Find : AC and XY « 14 cm. , 3 cm. »



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4 **Using the shown data in the figure , then prove that :**

Δ XYZ and Δ BCA are similar , then find the perimeter of Δ XYZ « 9 cm. »



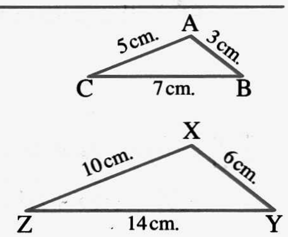
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5 **In the opposite figure :**

1 **Prove that :** Δ ABC and Δ XYZ are similar.

2 If : m (∠ B) + m (∠ C) = 60° ,

find : m (∠ X) « 120° »



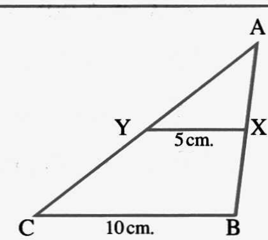
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6 **In the opposite figure :**

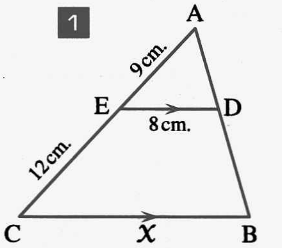
If Δ AXY ~ Δ ABC

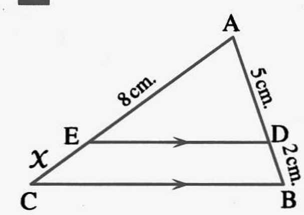
XY = 5 cm. and BC = 10 cm. ,

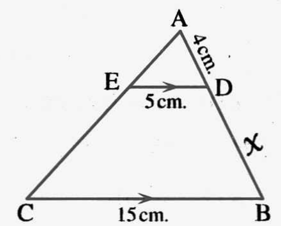
Prove that : **1** $\overline{XY}$ // $\overline{BC}$ **2** Y is the midpoint of $\overline{AC}$

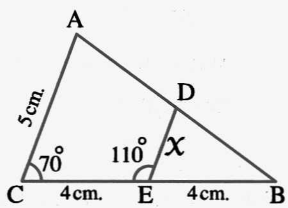


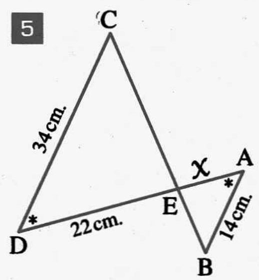
7 In each of the following , find the numerical value of x (Given that lengths are in cm.) :

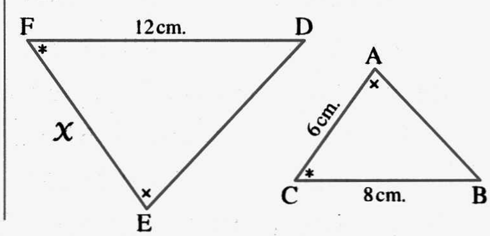
1


2


3


4


5


6


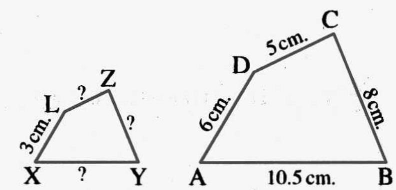
8 **In the opposite figure :**

The polygon ABCD ~ the polygon XYZL

If AB = 10.5 cm. , BC = 8 cm. , CD = 5 cm. ,

DA = 6 cm. and LX = 3 cm.

Find the length of each of : $\overline{XY}$, $\overline{YZ}$ and $\overline{ZL}$



« 5.25 cm. , 4 cm. , 2.5 cm. »

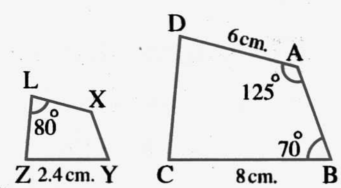
9 **In the opposite figure :**

If the figure ABCD ~ the figure XYZL

1 Calculate m (∠ BCD)

2 Calculate the length of $\overline{XL}$ and determine the enlargement ratio.

3 If the perimeter of the figure ABCD = 26 cm. , what is the perimeter of the figure XYZL



« 85° , 1.8 cm. , $\frac{10}{3}$, 7.8 cm. »

10 **In the opposite figure :**

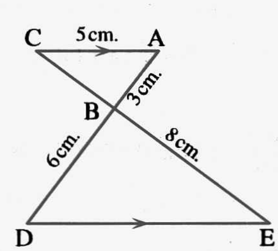
$\overline{AC} // \overline{ED}$, $\overline{AD} \cap \overline{CE} = \{B\}$

, AC = 5 cm. , BE = 8 cm. , AB = 3 cm. and BD = 6 cm.

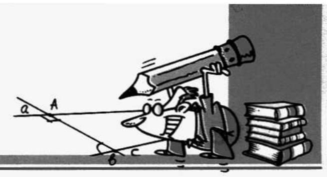
1 **Prove that :** Δ ABC ~ Δ DBE

2 **Find the length of each of :** $\overline{BC}$ and $\overline{ED}$

3 **Find :** the ratio of enlargement.



« 4 cm. , 10 cm. , 2 »



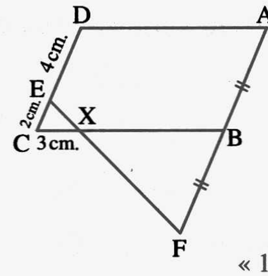
11 **In the opposite figure :**

ABCD is a parallelogram. B is the midpoint of $\overline{AF}$

CE = 2 cm. , DE = 4 cm. and XC = 3 cm.

Prove that : Δ ECX ~ Δ FBX

then find the length of : $\overline{AD}$



« 12 cm. »

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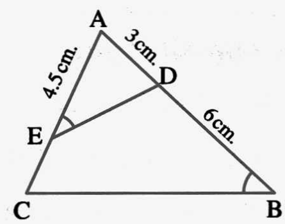
12 **In the opposite figure :**

m (∠ AED) = m (∠ B) , AD = 3 cm.

AE = 4.5 cm. and BD = 6 cm.

1 **Prove that :** Δ ADE ~ Δ ACB

2 **Find the length of :** $\overline{EC}$



« 1.5 cm. »

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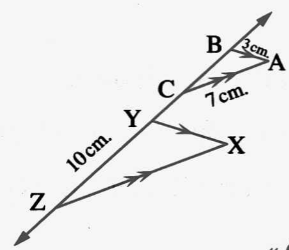
13 **In the opposite figure :**

$\overline{AB}$ // $\overline{XY}$, $\overline{AC}$ // $\overline{XZ}$

, AB = 3 cm. , AC = 7 cm. , YZ = 10 cm.

and XZ = 2 AC

Find : XY , BC



« 6 cm. , 5 cm. »

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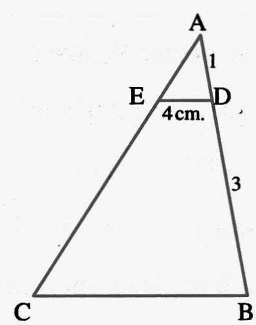
14 **In the opposite figure :**

Δ ADE ~ Δ ABC

, AD : DB = 1 : 3

1 **Find the ratio** $\frac{AE}{AC}$

2 **If** DE = 4 cm. , **find the length of :** $\overline{BC}$



« $\frac{1}{4}$, 16 cm. »

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15 **In the opposite figure :**

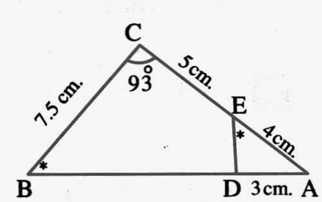
Δ ABC , D ∈ $\overline{AB}$, E ∈ $\overline{AC}$

, AE = 4 cm. , EC = 5 cm. , BC = 7.5 cm.

, AD = 3 cm. , m (∠ AED) = m (∠ B) and m (∠ C) = 93°

1 **Prove that :** Δ AED ~ Δ ABC

2 **Find the length of each of :** $\overline{BD}$ and m (∠ ADE)



« 9 cm. , 93° »

16 **In the opposite figure :**

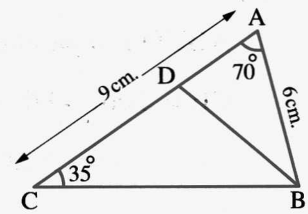
ABC is a triangle in which : m (∠ A) = 70°

, m (∠ C) = 35° , D ∈ $\overline{AC}$

If Δ ABD ~ Δ ACB

Find : m (∠ DBC) and if : AB = 6 cm. , AC = 9 cm.

Find the length of : $\overline{CD}$



« 40° , 5 cm. »

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17 **In the opposite figure :**

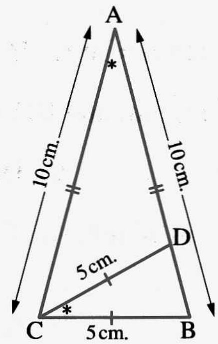
ABC is a triangle in which , D ∈ $\overline{AB}$

, AB = AC = 10 cm. , CD = CB = 5 cm.

and m (∠ DCB) = m (∠ BAC)

1 **Prove that :** Δ ABC ~ Δ CDB

2 **Find the length of :** $\overline{AD}$



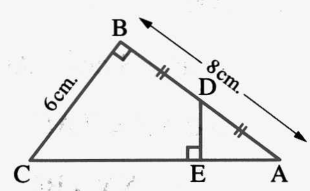
« 7.5 cm. »

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18 **In the opposite figure :**

ABC is a right-angled triangle at B , D is the midpoint of $\overline{AB}$, $\overline{DE} \perp \overline{AC}$, AB = 8 cm. , BC = 6 cm.

Find the length of : $\overline{DE}$



« 2.4 cm. »

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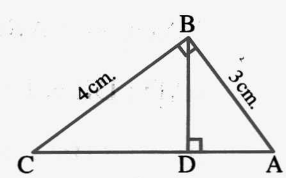
19 📖 **In the opposite figure :**

ABC is a right-angled triangle at B in which :

AB = 3 cm. , BC = 4 cm. and $\overline{BD} \perp \overline{AC}$

1 **Prove that :** Δ BAC ~ Δ DAB

2 **Find the length of each of :** $\overline{AD}$ and $\overline{DC}$



« 1.8 cm. , 3.2 cm. »

---

20 ABC is a triangle. $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ are bisected at D , E and F respectively

Prove that : Δ ABC ~ Δ EFD

---

21 📖 Two similar triangles, one of them has a perimeter of 74 cm. and the sides lengths of the other are 4.5 cm. , 6 cm. and 8 cm.

Find the length of the longest side in the first triangle. « 32 cm. »

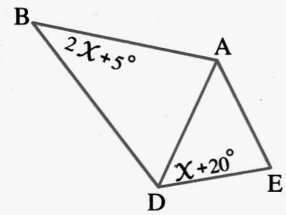


22 **In the opposite figure :**

Δ AED $\sim$ Δ ADB

, m ($\angle$ ADE) = $X + 20$ and m ($\angle$ ABD) = $2X + 5°$

Find : m ($\angle$ ADE)



« 35° »

23 **In the opposite figure :**

$\overline{BZ} \subset \overline{BC}$, $\overline{XZ} \parallel \overline{AC}$, $\overline{XY} \parallel \overline{AB}$

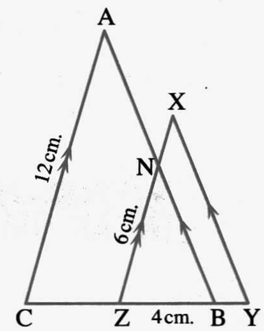
, $\overline{XZ} \cap \overline{AB} = \{N\}$, AC = 12 cm.

NZ = 3 NX = 6 cm. and BZ = 4 cm.

1 **Prove that :** Δ XYZ $\sim$ Δ NBZ $\sim$ Δ ABC

2 **Prove that :** Z is the midpoint of $\overline{BC}$

3 **Find the length of :** $\overline{YC}$



« $9\frac{1}{3}$ cm. »

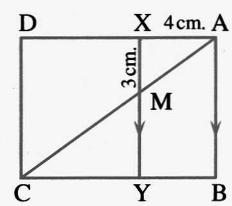
24 **In the opposite figure :**

ABCD is a rectangle in which AD = 12 cm. and X $\in \overline{AD}$ where AX = 4 cm. , $\overline{XY} \parallel \overline{AB}$ and intersects $\overline{AC}$ at M and $\overline{BC}$ at Y , where MX = 3 cm.

1 **Prove that :** Δ AMX $\sim$ Δ CMY

2 **Find the perimeter of :** Δ YMC

3 **Is the figure ABYM $\sim$ the figure CDXM ? Why ?**



« 24 cm. »

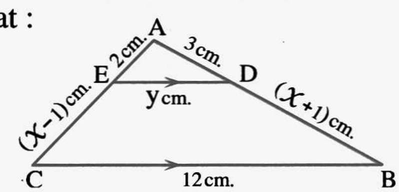
25 **In the opposite figure :**

ABC is a triangle in which D $\in \overline{AB}$ and E $\in \overline{AC}$ such that :

$\overline{DE} \parallel \overline{BC}$, AD = 3 cm. , AE = 2 cm. , BC = 12 cm.

, BD = (X + 1) cm. , EC = (X – 1) cm. and DE = y cm.

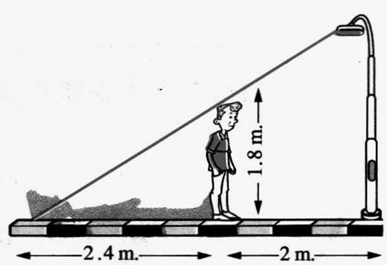
Find the length of each of : $\overline{AB}$, $\overline{EC}$ and $\overline{DE}$



« 9 cm. , 4 cm. , 4 cm. »

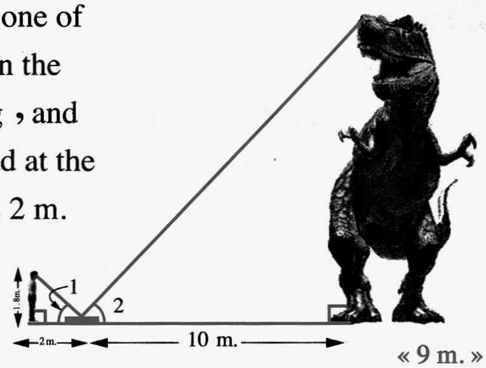
Life Applications

26 A man of height 1.8 m. stands in front of a lamppost at a distance of 2 m. from its base. If the length of the man's shade (when the lamppost is turned on) is 2.4 m. , find the height of the lamppost.



« 3.3 m. »

27 A man wanted to know the height of a dinosaur at one of the museums , then he put a mirror horizontally on the land at a distance of 10 m. from the dinosaur's leg , and he moved back till he could see the dinosaur's head at the mirror , then the distance that he moved back was 2 m. If the height of the man is 1.8 m. , what is the height of the dinosaur , given that : m (∠ 1) = m (∠ 2) ?



« 9 m. »

For excellent pupils

28 ABC and DEF are two triangles in which , AB = AC and DE = DF

If : $\frac{AB}{BC} = \frac{DE}{EF}$, prove that they are similar.

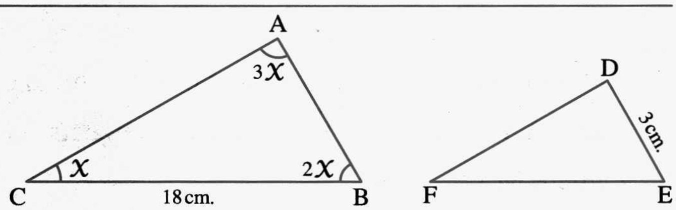
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29 **In the opposite figure :**

If Δ ABC ~ Δ DEF

, BC = 18 cm. and DE = 3 cm.

Find the length of : $\overline{EF}$



« 6 cm. »

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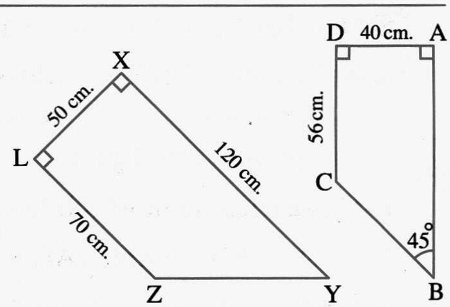
30 **In the opposite figure :**

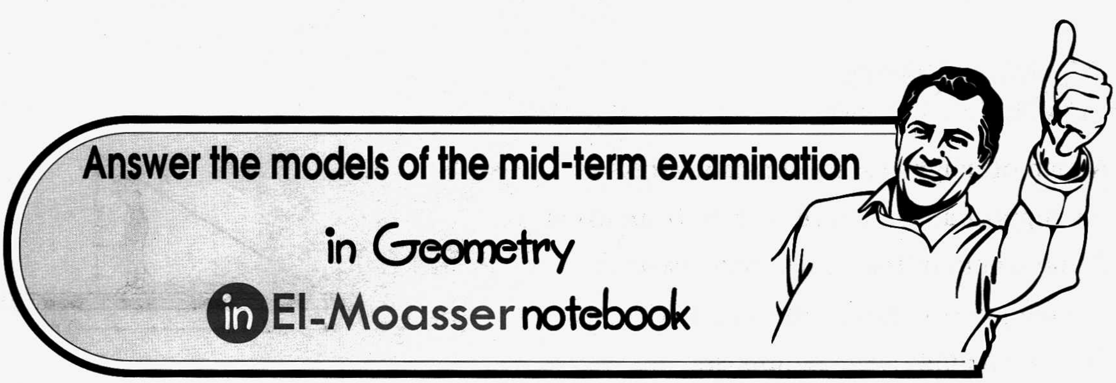
AD = 40 cm. , DC = 56 cm. , XL = 50 cm. ,

XY = 120° cm. , LZ = 70 cm. , m (∠B) = 45°

and m (∠ A) = m (∠ D) = m (∠ X) = m (∠ L) = 90°

Prove that : The polygon ABCD ~ the polygon XYZL







Worksheet 5 till lesson (5) unit (4)

Answer the following questions :

1 Choose the correct answer from the given ones :

(1) A rhombus whose diagonals lengths are 6 cm. and 8 cm. , then its area = ... cm^2

(a) 48 (b) 24 (c) 14 (d) 12

(2) The area of the square whose diagonal length is 4 cm. = ... cm^2

(a) 16 (b) 8 (c) 24 (d) 18

(3) The area of the rectangle whose dimensions are 3 cm. , 8 cm. .... the area of the triangle whose base length is 8 cm. and its corresponding height is 6 cm.

(a) < (b) > (c) = (d) ≠

(4) If the area of a parallelogram is 45 cm^2 and its height is 5 cm. , then the length of the corresponding base to this height = ... cm.

(a) 50 (b) 40 (c) 5 (d) 9

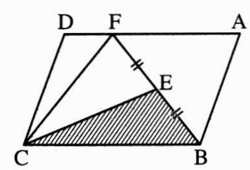
2 Complete the following :

(1) The trapezium in which the lengths of the two parallel bases are 5 cm. and 7 cm. and its height is 4 cm. , then its area = ... cm^2

(2) The median of a triangle divides its surface into ....

(3) Triangles of bases equal in length and lying between two parallel straight lines are ....

(4) **In the opposite figure :**

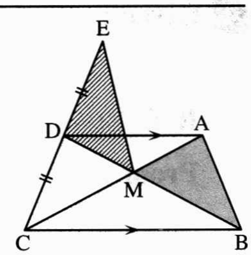


ABCD is a parallelogram ,

E is the midpoint of $\overline{BF}$,

then the area of Δ BEC = ... the area of the parallelogram ABCD

3 In the opposite figure :



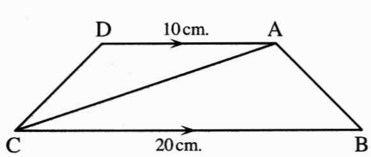
$\overline{AD} \parallel \overline{BC}$, $\overline{AC} \cap \overline{BD} = \{M\}$,

$E \in \overrightarrow{CD}$

such that : CD = DE

Prove that : the area of Δ MDE = the area of Δ AMB

4 In the opposite figure :



ABCD is a trapezium in which

$\overline{AD} \parallel \overline{BC}$, AD = 10 cm. , BC = 20 cm. ,

the area of Δ ABC = 40 cm^2

Find by proof : the area of the trapezium ABCD



Worksheet 6 till lesson (1) unit (5)

Answer the following questions :

1 Choose the correct answer from the given ones :

(1) If the ratio between the side lengths of two squares is one and if the perimeter of one of them is 8 cm. , then the area of the other square equals .... cm^2

(a) 64 (b) 2 (c) 8 (d) 4

(2) The lengths of the two parallel bases of a trapezium are 14 cm. and 10 cm. and its area is 120 cm^2 , then its height = ... cm.

(a) 5 (b) 10 (c) 20 (d) 30

(3) If the ratio of magnification between two similar triangles is 3 : 2 and the length of a side from the greatest triangle = 15 cm. , then the length of the corresponding side in the smallest triangle = ... cm.

(a) 5 (b) 10 (c) 6 (d) 9

(4) If the area of a square is 18 cm^2 , then its diagonal length = ... cm.

(a) 36 (b) 12 (c) 9 (d) 6

2 Complete the following :

(1) If the lengths of two adjacent sides in a parallelogram are 8 cm. and 16 cm. and its greatest height = 5 cm. , then its area = ... cm^2

(2) A rhombus , the lengths of its two diagonals are X cm. and y cm. , then its area = ... cm^2

(3) Two triangles are similar if the lengths of their corresponding sides are .... or the measures of their corresponding angles are ....

(4) ABCD is a parallelogram , E $\in \overleftrightarrow{CD}$, if the area of Δ AEB = 20 cm^2 , then the area of ▱ ABCD = ... cm^2

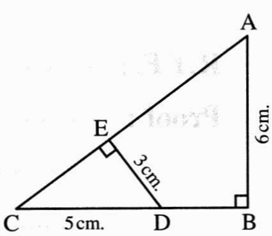
3 In the opposite figure :

Δ ABC is right-angled at B ,

$\overline{DE} \perp \overline{AC}$, AB = 6 cm. , ED = 3 cm. , CD = 5 cm.

Prove that : Δ CED $\sim$ Δ CBA

and find the length of $\overline{AC}$ and the length of $\overline{BD}$



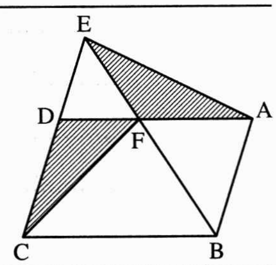
4 In the opposite figure :

ABCD is a parallelogram ,

E $\in \overrightarrow{CD}$, $\overline{AD} \cap \overline{BE}$ = {F}

Prove that :

the area of Δ AFE = the area of Δ CDF



$\therefore$ the area of Δ XBC = $\frac{1}{3}$ the area of Δ ABC

Similarly we can prove that :

the area of Δ YBC = $\frac{1}{3}$ the area of Δ ABC

$\therefore$ the area of Δ XBC = the area of Δ YBC but they have the common base $\overline{BC}$ and on one side of it.

$\therefore \overline{XY} // \overline{BC}$ (Q.E.D.)

Answers of Exercise 5

1

1. its height , the lengths of its diagonals
2. its side , the square of the length of its diagonal
3. $\frac{1}{2}$ of the sum of lengths of its two parallel bases
4. its height, the middle base.
5. Congruent (equal in measure)
6. Congruent (equal in length)

2

1. The area = $6 \times 5 = 30$ cm^2.
2. The area = $12 \times 8 = 96$ cm^2.
3. The area = $\frac{1}{2} \times 8 \times 10 = 40$ cm^2.
4. The area = $\frac{1}{2} \times 24 \times 10 = 120$ cm^2.
5. The area = $\frac{1}{2} \times 10 \times 10 = 50$ cm^2.
6. The area = $\frac{1}{2} \times 8 \times 8 = 32$ cm^2.
7. The area = $\frac{(6+8)}{2} \times 12 = 84$ cm^2.
8. The area = $\frac{(8+10)}{2} \times 5 = 45$ cm^2.
9. The area = $7 \times 6 = 42$ cm^2.
10. The area = $12 \times 8 = 96$ cm^2.

3

1 a	2 c	3 a	4 b	5 b
6 c	7 b	8 b	9 c	10 b

4

The area of the rectangle = $2 \times 9 = 18$ cm^2.

The area of the square = 18 cm^2

$\therefore \frac{1}{2}$ (the length of the diagonal)$^2 = 18$

$\therefore$ (the length of the diagonal)$^2 = 36$

$\therefore$ the length of the diagonal = 6 cm.

5

$\because$ The area of the rhombus = $\frac{1}{2} \times 8 \times 16 = 64$ m^2.

$\therefore$ The area of the square = 64 m^2.

$\therefore$ The side length of the square = 8 m.

$\therefore$ The perimeter of the square = $8 \times 4 = 32$ m.

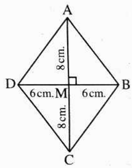
6

$\because$ The area of the rhombus = $\frac{1}{2} \times 18 \times 24 = 216$ m^2.

$\therefore$ The area of the trapezium = 216 m^2.

$\therefore$ The length of the middle base = $\frac{216}{12} = 18$ m.

7



From the figure :

$(AB)^2 = (6)^2 + (8)^2 = 100$

$\therefore$ AB = 10 cm.

$\therefore$ The side length = 10 cm.

$\therefore$ The area of the rhombus

$= \frac{1}{2} \times 12 \times 16 = 96$ cm^2.

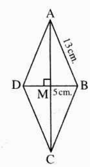
$\therefore$ The height = $\frac{96}{10} = 9.6$ cm.

8

$\because$ The perimeter of the rhombus = 52 cm.

$\therefore$ The side length of the rhombus = $\frac{52}{4} = 13$ cm.

Drawing the rhombus as shown in the figure such that BD = 10 cm.



$\therefore$ BM = 5 cm.

$\because \overline{AC} \perp \overline{BD}$

$\therefore (AM)^2 = (AB)^2 - (BM)^2$

$= 169 - 25 = 144$

$\therefore$ AM = 12 cm.

$\therefore$ AC = 24 cm.

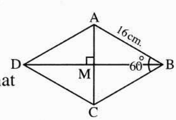
The area of the rhombus = $\frac{1}{2} \times 10 \times 24 = 120$ cm^2.

9

$\because$ The perimeter of the rhombus = 64 cm.

$\therefore$ The side length of the rhombus = $\frac{64}{4} = 16$ cm.

Drawing the rhombus as shown in the figure such that AB = 16 cm.



, m ($\angle$ B) = 60°

$\because$ The diagonal of the rhombus bisects the two angles joinning their vertices.

$\therefore$ m ($\angle$ ABM) = 30°

$\because$ The diagonals of the rhombus are perpendicular

$\therefore$ m ($\angle$ AMB) = 90°

$\therefore$ In the right-angled triangle, the length of the side opposite to the angle of measure 30° = $\frac{1}{2}$ the length of the hypotenuse.

$\therefore$ AM = $\frac{1}{2} \times 16 = 8$ cm.

$\therefore$ AC = $2 \times 8 = 16$ cm.

$\because$ m ($\angle$ AMB) = 90°

$\therefore (BM)^2 = (AB)^2 - (AM)^2$

$\therefore (BM)^2 = 256 - 64 = 192$

$\therefore BM = \sqrt{192} = 8\sqrt{3}$

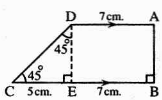
$\therefore BD = 2 \times 8\sqrt{3} = 16\sqrt{3}$ cm.

$\therefore$ The area of the rhombus = $\frac{1}{2} \times 16 \times 16\sqrt{3}$
= $128\sqrt{3}$ cm^2.

10

1 Construction :

Draw $\overline{DE} \perp \overline{BC}$



Proof :

$\because$ m ($\angle$ C) = 45°

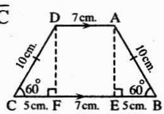
$\therefore$ m ($\angle$ CDE) = 180° – (90° + 45°) = 45°

$\therefore$ DE = EC = 5 cm.

$\therefore$ The area of the trapezium = $\frac{1}{2}$ (7 + 12) × 5
= 47.5 cm^2.

2 Construction :

Draw $\overline{AE} \perp \overline{BC}$, $\overline{DF} \perp \overline{BC}$



Proof :

$\because$ m ($\angle$ CDF)
= 180° – (90° + 60°) = 30°

$\because$ In the right - angled triangle, the length of the side opposite to the angle of measure 30°
= $\frac{1}{2}$ the length of the hypotenuse.

$\therefore$ FC = 5 cm.

$\therefore (DF)^2 = (DC)^2 - (FC)^2 = 100 - 25 = 75$

$\therefore DF = 5\sqrt{3}$ cm. similarly BE = 5 cm.

$\therefore$ BC = 5 + 7 + 5 = 17 cm.

$\therefore$ The area of the trapezium
= $\frac{1}{2}(7 + 17) \times 5\sqrt{3} = \frac{1}{2} \times 24 \times 5\sqrt{3} = 60\sqrt{3}$ cm^2.

11

Let the length of the smallest diagonal be $3x$ cm.

$\because$ The length of the greatest diagnoal = $4x$ cm.

$\therefore 3x = 9$ $\therefore x = 3$

$\therefore$ The length of the greatest diagonal = 4 × 3 = 12 cm.

$\therefore$ The area of the rhombus = $\frac{1}{2} \times 9 \times 12 = 54$ cm^2.

12

Let the length of the smallest diagonal be $5x$ cm.

$\because$ The length of the greatest diagonal = $8x$ cm.

$\therefore$ The area = $\frac{1}{2} \times 5x \times 8x = 20x^2$

$\therefore 20x^2 = 2000$ $\therefore x^2 = 100$ $\therefore x = 10$ cm.

$\therefore$ The lengths of the two diagonals are 50 cm. and 80 cm.

13

Let the lengths of the two bases be $2x$ cm. and $3x$ cm.

$\because \frac{1}{2}(2x + 3x) = 30$ $\therefore 5x = 60$ $\therefore x = 12$

$\therefore$ The lengths of the two bases are 24 cm. and 36 cm.

The area of the trapezium = 30 × 24 = 720 cm^2.

14

Let the lengths of the two parallel bases be $3x$ cm. and $2x$ cm.

$\because$ The area = $\frac{1}{2}(3x + 2x) \times 12$

$\therefore \frac{1}{2}(3x + 2x) \times 12 = 180$

$\therefore 30x = 180$ $\therefore x = 6$ cm.

$\therefore$ The lengths of the two bases are 18 cm. and 12 cm.

15

Let the lengths of the two parallel bases and the height be $3x$ cm., $2x$ cm. and $4x$ cm.

$\because$ The area = $\frac{1}{2}(3x + 2x) \times 4x$

$\therefore 10x^2 = 4000$ $\therefore x^2 = 400$ $\therefore x = 20$ cm.

$\therefore$ The lengths of the two parallel bases are 60 cm, 40 cm.

$\therefore$ The length of the middle base = $\frac{60 + 40}{2} = 50$ cm.

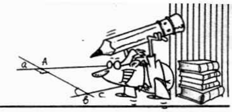
16

the area of the piece of land which is in the shape of a trapezium = $\frac{1}{2}(76 + 64) \times 45 = 3150$ m^2.
and the area of the other piece of land which is in the shape of a rhombus = $\frac{1}{2} \times 74 \times 90 = 3330$ m^2.

$\therefore$ The area of the rectangular piece of land
= 3150 + 3330 = 6480 m^2.

Let the length of the rectangular piece be $5x$

$\therefore$ Its width = $4x$ m. $\therefore 5x \times 4x = 6480$

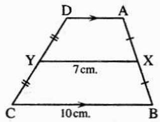


$\therefore x^2 = 324$ $\qquad \therefore x = \sqrt{324} = 18$ m.

$\therefore$ The length $= 5 \times 18 = 90$ m.

The width $= 4 \times 18 = 72$ m.

17



$\because \frac{AD + BC}{2} = XY$

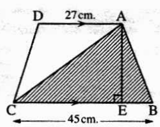
$\therefore \frac{AD + 10}{2} = 7$

$\therefore AD = 4$ cm.

$\therefore$ The area = the length of the middle base $\times$ the perpendicular distance between the two parallel bases $\overline{AD}$ and $\overline{BC}$

$\therefore$ The perpendicular distance between the two parallel bases $\overline{AD}$ and $\overline{BC} = \frac{35}{7} = 5$ cm.

18



$\because$ the area of Δ ABC

$= \frac{1}{2}$ BC $\times$ AE

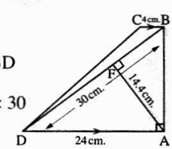
$\therefore 225 = \frac{1}{2} \times 45 \times$ AE

$\therefore$ AE = 10 cm.

The area of the trapezium

$= \frac{1}{2} (27 + 45) \times 10 = 360$ cm^2.

19



$\because$ the area of Δ ABD

$= \frac{1}{2}$ AB $\times$ AD $= \frac{1}{2}$ AF $\times$ BD

$= \frac{1}{2}$ AB $\times 24 = \frac{1}{2} \times 14.4 \times 30$

$\therefore$ AB = 18 cm.

The area of the trapezium ABCD

$= \frac{4 + 24}{2} \times 18 = 252$ cm^2.

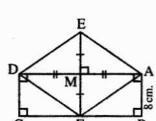
20

The area of the figure ABCDE = the area of Δ ALE + the area of Δ ELD + the area of Δ AMB + the area of Δ CDN + the area of the trapezium BCNM $= \frac{1}{2} \times 2 \times 3$

$+ \frac{1}{2} \times 8 \times 3 + \frac{1}{2} \times 5.6 \times 4 + \frac{1}{2} \times 3 \times 4 + \frac{5.6 + 4}{2} \times 3$

$= 3 + 12 + 11.2 + 6 + 14.4 = 46.6$ cm^2.

21



$\because$ ABCD is a rectangle ,

$\overline{MF} \perp \overline{AD}$

$\therefore$ AB = MF = 8 cm.

$\because$ M is the midpoint of $\overline{EF}$

$\therefore$ EF $= 2 \times 8 = 16$ cm. (1)

$\therefore$ the area of the rectangle ABCD = AB $\times$ BC

$\therefore 144 = 8 \times$ BC

$\therefore$ BC $= \frac{144}{8} = 18$ cm.

$\therefore$ AD = 18 cm. (2)

$\therefore$ In the figure AFDE : the two diagonals $\overline{AD}$ and $\overline{EF}$ bisect each other , $\overline{FE} \perp \overline{AD}$

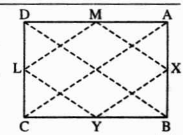
$\therefore$ The figure AFDE is a rhombus.

From (1) and (2) we find that :

The area of the figure AFDE $= \frac{1}{2}$ EF $\times$ AD

$= \frac{1}{2} \times 16 \times 18 = 144$ cm^2.

22



1 $\because$ X and Y are the midpoints of $\overline{AB}$ and $\overline{BC}$

$\therefore \overline{XY} \,//\, \overline{AC}$

$\therefore$ XY $= \frac{1}{2}$ AC (1)

$\therefore$ L and M are the midpoints of $\overline{DC}$ and $\overline{DA}$

$\therefore \overline{LM} \,//\, \overline{AC}$, LM $= \frac{1}{2}$ AC (2)

From (1) and (2) :

$\therefore \overline{LM} \,//\, \overline{XY}$, LM = XY

$\therefore$ XYLM is a parallelogram. (3)

$\because$ XY $= \frac{1}{2}$ AC , XM $= \frac{1}{2}$ BD

, but AC = DB (two diagonals in the rectangle ABCD)

$\therefore$ XY = XM (4)

From (3) and (4) :

$\therefore$ The figure XYLM is a rhombus.

$\therefore$ The area of the rhombus

$= \frac{1}{2}$ the product of the lengths of its two diagonals

$= \frac{1}{2}$ XL $\times$ YM $= \frac{1}{2} \times 8 \times 6 = 24$ cm^2.

2 $\therefore (AC)^2 = (AB)^2 + (BC)^2$

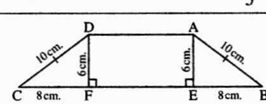
$\therefore (AC)^2 = 36 + 64 = 100$

$\therefore$ AC = 10 cm. $\qquad \because$ XY $= \frac{1}{2}$ AC

$\therefore$ XY $= \frac{1}{2} \times 10 = 5$ cm.

$\therefore$ the height of the rhombus XYLM $= \frac{24}{5} = 4.8$ cm.

23



$\because$ The area of the trapezium = 120 cm^2 and the length of the middle base = 20 cm.

$\therefore$ Its height = $\frac{120}{20}$ = 6 cm.

$\therefore$ The perimeter of the trapezium = 60 cm. and the length of the middle base = 20 cm.

$\therefore$ AD + BC = 2 × 20 = 40 cm.

$\therefore$ AB + DC = 60 – 40 = 20 cm.

$\therefore$ AB = DC = $\frac{20}{2}$ = 10 cm.

From the figure :

$\therefore$ $(BE)^2 = (AB)^2 - (AE)^2 = 100 - 36 = 64$

$\therefore$ BE = 8 cm. , similarly FC = 8 cm.

$\because$ AD = FE , $\because$ AD + BC = 40 cm.

$\therefore$ AD + FE + 8 + 8 = 40 $\therefore$ 2 AD = 40 – 16

$\therefore$ 2 AD = 24 $\therefore$ AD = 12 cm.

$\therefore$ BC = 12 + 8 + 8 = 28 cm.

24

Let BD = 5 X cm. , AC = 6 X cm.

$\because$ $5X + 6X = 33$ $\therefore$ $11X = 33$ $\therefore$ $X = 3$

$\therefore$ BD = 15 cm. , AC = 18 cm.

$\therefore$ ME = $\frac{2}{3}$ MA $\therefore$ ME = $\frac{2}{3} \times 9$ = 6 cm.

the area of Δ EBD = $\frac{1}{2}$ BD × EM

= $\frac{1}{2} \times 15 \times 6$ = 45 cm^2 (1)

the area of the rhombus ABCD = $\frac{1}{2}$ BD × AC

= $\frac{1}{2} \times 15 \times 18$ = 135 cm^2 (2)

From (1) and (2) :

$\therefore$ The area of the shaded part = 135 – 45 = 90 cm^2

Answers of the general exercises on unit four

First : Completion questions

1. 30
2. equal
3. 40 cm^2
4. their vertices lie on a straight line parallel to this base.
5. one is carrying this base are equal in area.
6. two triangular surfaces equal in area.
7. base length × its corresponding height.
8. area
9. $\frac{1}{2} X$ y cm.
10. 24 cm^2
11. equal
12. 6 cm.
13. 7 cm.
14. 20 cm^2
15. 10
16. 12 cm.
17. 6

Second : Multiple choice questions

1. b
2. c
3. a
4. d
5. a
6. c
7. c
8. d
9. c
10. a
11. b
12. d
13. b

Third : Essay questions

1 15 cm^2

2 prove by yourself

3 12 cm.

4 , **5** Prove by yourself

6 24 cm.

7 : **9** Prove by yourself

10 $4\frac{1}{2}$ cm.

Answers of Unit Five

Answers of Exercise 6

1

1. angles
2. side lengths
3. similar
4. side lengths.
5. similar
6. equal in measure , proportional
7. congruent
8. 3 : 4

2

1. b
2. d
3. d
4. b
5. d
6. b
7. c

3

$\because \Delta ABC \sim \Delta XYZ$ $\therefore \frac{AC}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$

$\therefore \frac{6}{XY} = \frac{10}{5} = \frac{AC}{7}$ $\therefore AC = \frac{7 \times 10}{5} = 14$ cm.

, $XY = \frac{5 \times 6}{10} = 3$ cm. (the req.)

4

In $\Delta\Delta$ ABC and ZXY :

$\because m(\angle A) = m(\angle Z)$, $m(\angle C) = m(\angle Y)$

$\therefore m(\angle B) = m(\angle X)$ $\therefore \Delta XYZ \sim \Delta BCA$

$\therefore \frac{XY}{BC} = \frac{YZ}{CA}$ $\therefore \frac{XY}{16} = \frac{3}{12}$

$\therefore XY = \frac{3 \times 16}{12} = 4$ cm.

$\therefore$ The perimeter of $\Delta XYZ = 2 + 3 + 4 = 9$ cm

(the req.)

5

$\because \frac{AB}{XY} = \frac{3}{6} = \frac{1}{2}$, $\frac{AC}{XZ} = \frac{5}{10} = \frac{1}{2}$, $\frac{BC}{YZ} = \frac{7}{14} = \frac{1}{2}$

$\therefore \frac{AB}{XY} = \frac{AC}{XZ} = \frac{BC}{YZ}$

$\therefore \Delta ABC \sim \Delta XYZ$ (the first req.)

$\because m(\angle B) + m(\angle C) = 60°$

$\therefore m(\angle A) = 120°$

$\therefore m(\angle A) = m(\angle X)$

$\therefore m(\angle X) = 120°$ (the second req.)

6

$\because \Delta AXY \sim \Delta ABC$

$\therefore m(\angle AXY) = m(\angle ABC)$

but they are corresponding.

$\therefore \overline{XY} \parallel \overline{BC}$ (Q.E.D. 1)

$\because \Delta AXY \sim \Delta ABC$

$\therefore \frac{AY}{AC} = \frac{XY}{BC}$ $\therefore \frac{AY}{AC} = \frac{1}{2}$

$\therefore$ Y is the midpoint of $\overline{AC}$ (Q.E.D. 2)

7

1. $\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{DB}$ is a transversal to them

$\therefore m(\angle B) = m(\angle ADE)$ (corresponding angles)

Similarly $m(\angle C) = m(\angle AED)$

(corresponding angles)

$\angle A$ is a common angle.

$\therefore \Delta ABC \sim \Delta ADE$ $\therefore \frac{DE}{BC} = \frac{AE}{AC}$

$\therefore \frac{8}{x} = \frac{9}{21}$ $\therefore x = \frac{8 \times 21}{9} = 18\frac{2}{3}$ cm.

2. $\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{DB}$ is a transversal to them.

$\therefore m(\angle B) = m(\angle ADE)$ (corresponding angles)

Similarly $m(\angle C) = m(\angle AED)$

(corresponding angles)

$\angle A$ is a common angle.

$\therefore \Delta ABC \sim \Delta ADE$

$\therefore \frac{AB}{AD} = \frac{AC}{AE}$ $\therefore \frac{7}{5} = \frac{AC}{8}$

$\therefore AC = \frac{7 \times 8}{5} = 11.2$ cm.

$\therefore x = 11.2 - 8 = 3.2$ cm.

3. $m(\angle B) = m(\angle ADE)$ (corresponding angles)

, $m(\angle C) = m(\angle AED)$ (corresponding angles)

, $\angle A$ is common in $\Delta\Delta$ ADE and ABC

$\therefore \Delta ADE \sim \Delta ABC$

$\therefore \frac{AD}{AB} = \frac{DE}{BC}$ $\therefore \frac{4}{x+4} = \frac{5}{15}$ $\therefore \frac{4}{x+4} = \frac{1}{3}$

$\therefore x + 4 = 12$ $\therefore x = 8$ cm.

4. $m(\angle DEB) = 180° - 110° = 70°$

$\therefore$ In $\Delta\Delta$ DEB , ACB :

$m(\angle DEB) = m(\angle C)$, $\angle B$ is common

$\therefore m(\angle BDE) = m(\angle A)$ $\therefore \Delta DEB \sim \Delta ACB$

$\therefore \frac{DE}{AC} = \frac{EB}{CB}$ $\therefore \frac{x}{5} = \frac{4}{8}$

$\therefore x = \frac{5 \times 4}{8} = 2\frac{1}{2}$ cm.

5. In $\Delta\Delta$ ABE and DCE

$m(\angle A) = m(\angle D)$, $m(\angle AEB) = m(\angle DEC)$ (V.O.A)

$\therefore m(\angle B) = m(\angle C)$ $\therefore \Delta ABE \sim \Delta DCE$

$\therefore \frac{AB}{DC} = \frac{AE}{DE}$ $\therefore \frac{14}{34} = \frac{x}{22}$

$\therefore x = \frac{14 \times 22}{34} = 9\frac{1}{17}$ cm.

6 In ΔΔ ABC and EDF :

$\because$ m (∠ A) = m (∠ E) , m (∠ C) = m (∠ F)

$\therefore$ m (∠ B) = m (∠ D)

$\therefore \Delta ABC \sim \Delta EDF \qquad \therefore \frac{AC}{EF} = \frac{BC}{DF}$

$\therefore \frac{6}{x} = \frac{8}{12} \qquad \therefore x = \frac{6 \times 12}{8} = 9$ cm.

8

$\because$ The polygon ABCD ~ the polygon XYZL

$\therefore \frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZL} = \frac{DA}{LX} \qquad \therefore \frac{10.5}{XY} = \frac{8}{YZ} = \frac{5}{ZL} = \frac{6}{3}$

$\therefore XY = \frac{3 \times 10.5}{6} = 5.25$ cm. (the first req.)

$, YZ = \frac{8 \times 3}{6} = 4$ cm. (the second req.)

$, ZL = \frac{5 \times 3}{6} = 2.5$ cm. (the third req.)

9

$\because$ The figure ABCD ~ the figure XYZL

$\therefore$ m (∠ D) = m (∠ L) $\qquad \therefore$ m (∠ D) = 80°

m (∠ BCD) = 360° – (125° + 70° + 80°) = 85°

(the first req.)

$\because \frac{AD}{XL} = \frac{BC}{YZ} \qquad \therefore \frac{6}{XL} = \frac{8}{2.4}$

$\therefore XL = \frac{2 \times 2.4}{4} = 1.8$ cm. (the second req.)

$\therefore$ The ratio of enlargement $= \frac{8}{2.4} = \frac{10}{3}$

(the third req.)

$\therefore \frac{\text{The perimeter of the figure ABCD}}{\text{The perimeter of the figure XYZL}}$

= the ratio of enlargement

$\therefore \frac{26}{\text{The perimeter of the figure XYZL}} = \frac{10}{3}$

$\therefore$ The perimeter of the figure XYZL

$= \frac{26 \times 3}{10} = 7.8$ cm. (the fourth req.)

10

$\because \overline{AC} // \overline{ED}$, $\overleftrightarrow{AD}$ is a transversal to them.

$\therefore$ m (∠ A) = m (∠ D) (alternate angles) (1)

$\because \overline{AC} // \overline{ED}$, $\overleftrightarrow{CE}$ is a transversal to them.

$\therefore$ m (∠ C) = m (∠ E) (alternate angles) (2)

$\because$ m (∠ ABC) = m (∠ EBD) (V.O.A) (3)

From (1) , (2) , (3) :

$\therefore \Delta ABC \sim \Delta DBE$ (the first req.)

$\therefore \frac{AB}{DB} = \frac{BC}{BE} = \frac{CA}{ED} \qquad \therefore \frac{3}{6} = \frac{BC}{8} = \frac{5}{ED}$

$\therefore$ BC = 4 cm. , DE = 10 cm. (the second req.)

the ratio of enlargement $= \frac{DB}{AB} = \frac{6}{3} = 2$

(the third req.)

11

$\because \overline{AF} // \overline{DC}$, $\overleftrightarrow{BC}$ is a transversal to them.

$\therefore$ m (∠ FBC) = m (∠ C) (alternate angles)

$\because \overline{AF} // \overline{DC}$, $\overleftrightarrow{FE}$ is a transversal to them.

$\therefore$ m (∠ F) = m (∠ XEC) (alternate angles)

$\because$ m (∠ BXF) = m (∠ CXE) (V.O.A)

$\therefore \Delta ECX \sim \Delta FBX$ (the first req.)

$\because$ AB = DC (properties of ▱) $\qquad \therefore$ AB = 6 cm.

$\because$ B is the midpoint of $\overline{AF}$

$\therefore$ AB = BF = 6 cm.

$\because \frac{EC}{FB} = \frac{CX}{BX} \qquad \therefore \frac{2}{6} = \frac{3}{BX} \qquad \therefore$ BX = 9 cm.

$\therefore$ BC = 12 cm. $\qquad \because$ BC = AD (properties of ▱)

$\therefore$ AD = 12 cm. (the second req.)

12

In Δ Δ ABC , AED :

$\because$ m (∠ B) = m (∠ AED) , ∠ A is a common angle

$\therefore$ m (∠ C) = m (∠ ADE)

$\therefore \Delta ABC \sim \Delta AED$ (the first req.)

$\therefore \frac{AD}{AC} = \frac{AE}{AB} \qquad \therefore \frac{3}{AC} = \frac{4.5}{9}$

$\therefore AC = \frac{3 \times 9}{4.5} = 6$ cm. $\qquad \therefore$ EC = 6 – 4.5 = 1.5 cm.

(the second req.)

13

$\because \overline{AB} // \overline{XY}$, $\overleftrightarrow{BC}$ is a transversal to them.

$\therefore$ m (∠ ABC) = m (∠ XYZ) (corresponding angles) (1)

$\because \overline{AC} // \overline{XZ}$, $\overleftrightarrow{BC}$ is a trarsversal to them.

$\therefore$ m (∠ ACB) = m (∠ XZY) (corresponding angles) (2)

$\therefore$ In Δ Δ ABC , XYZ :

m (∠ A) = m (∠ X) (3)

From (1) , (2) and (3) : $\qquad \therefore \Delta ABC \sim \Delta XYZ$

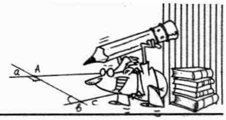
$\therefore \frac{AB}{XY} = \frac{BC}{YZ} = \frac{CA}{ZX} \qquad \therefore \frac{3}{XY} = \frac{BC}{10} = \frac{7}{14}$ (XZ = 2 AC)

$\therefore$ XY = 6 cm. , BC = 5 cm. (the req.)

14

$\because \Delta ADE \sim \Delta ABC \qquad \therefore \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$

$\because \frac{AD}{DB} = \frac{1}{3} \qquad \therefore \frac{AD}{AB} = \frac{1}{4}$



$\therefore \frac{AE}{AC} = \frac{1}{4}$ (the first req.)

$\therefore \frac{DE}{BC} = \frac{AE}{AC}$ $\therefore \frac{4}{BC} = \frac{1}{4}$

$\therefore BC = 16$ cm. (the second req.)

15

In $\Delta\,\Delta$ AED , ABC :

$\because m(\angle AED) = m(\angle B)$, $\angle A$ is a common angle.

$\therefore m(\angle ADE) = m(\angle C)$

$\therefore \Delta AED \sim \Delta ABC$ (the first req.)

$\therefore \frac{AE}{AB} = \frac{AD}{AC}$ $\therefore \frac{4}{AB} = \frac{3}{9}$ $\therefore AB = 12$ cm.

$\therefore BD = 12 - 3 = 9$ cm.

$m(\angle ADE) = m(\angle C) = 93°$ (the second req.)

16

$\because \Delta ABD \sim \Delta ACB$

$\therefore m(\angle DBA) = m(\angle BCA)$

$\therefore m(\angle DBA) = 35°$ (1)

In Δ ABC :

$\because m(\angle ABC) = 180° - (70° + 35°) = 75°$ (2)

From (1) , (2) :

$\therefore m(\angle DBC) = 75° - 35° = 40°$ (the first req.)

$\frac{AB}{AC} = \frac{BD}{CB} = \frac{AD}{AB}$ $\therefore \frac{6}{9} = \frac{AD}{6}$ $\therefore AD = 4$ cm.

$\therefore CD = 9 - 4 = 5$ cm. (the second req.)

17

In $\Delta\,\Delta$ ABC , CDB :

$m(\angle A) = m(\angle BCD)$ (1)

In Δ ABC : $\because AB = AC$

$\therefore m(\angle DBC) = m(\angle ACB)$ (2)

In Δ BCD : $\because CB = CD$

$\therefore m(\angle BDC) = m(\angle CBA)$ (3)

From (1) , (2) and (3) :

$\therefore \Delta ABC \sim \Delta CDB$ (the first req.)

$\therefore \frac{AB}{CD} = \frac{BC}{DB}$ $\therefore \frac{10}{5} = \frac{5}{DB}$ $\therefore DB = 2.5$ cm.

$\therefore AD = 10 - 2.5 = 7.5$ cm. (the second req.)

18

In Δ ABC : $\because m(\angle B) = 90°$

$\because (AC)^2 = (AB)^2 + (BC)^2 = 64 + 36 = 100$

$\therefore AC = 10$ cm.

, $\because$ D is the midpoint of $\overline{AB}$

$\therefore AD = DB = 4$ cm.

In $\Delta\Delta$ AED , ABC :

$m(\angle AED) = m(\angle B) = 90°$ (given)

, $\angle A$ is common

$\therefore m(\angle ADE) = m(\angle ACB)$

$\therefore \Delta AED \sim \Delta ABC$

$\therefore \frac{DE}{CB} = \frac{AD}{AC}$ $\therefore \frac{DE}{6} = \frac{4}{10}$

$\therefore DE = \frac{6 \times 4}{10} = 2.4$ cm. (The req.)

19

In $\Delta\,\Delta$ BAC , DAB :

$m(\angle ABC) = m(\angle ADB) = 90°$

, $\angle A$ is a common angle. $\therefore m(\angle C) = m(\angle ABD)$

$\therefore \Delta BAC \sim \Delta DAB$ (the first req.)

$\because \Delta$ ABC is right-angled at B

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 9 + 16 = 25$

$\therefore AC = 5$ cm.

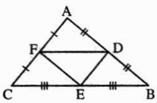
$\therefore \frac{AD}{AB} = \frac{AB}{AC} = \frac{BD}{CB}$ $\therefore \frac{AD}{3} = \frac{3}{5} = \frac{BD}{4}$

$\therefore AD = \frac{3 \times 3}{5} = 1.8$ cm. (the second req.)

$\because DC = AC - AD$

$\therefore DC = 5 - 1.8 = 3.2$ cm. (the third req.)

20



$\because$ D is the midpoint of $\overline{AB}$

F is the midpoint of $\overline{AC}$

$\therefore DF = \frac{1}{2} BC$

$\therefore \frac{DF}{BC} = \frac{1}{2}$

similarly : $\frac{EF}{AB} = \frac{1}{2}$, $\frac{DE}{AC} = \frac{1}{2}$

$\therefore \frac{DF}{BC} = \frac{EF}{AB} = \frac{DE}{AC}$

$\therefore \Delta ABC \sim \Delta EFD$ (Q.E.D.)

21

Assuming that the triangle whose side lengths are given be ABC and the other is XYZ

$\because$ The two triangles are similar.

$\therefore \frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ} = \frac{\text{The perimeter of } \Delta ABC}{\text{The perimeter of } \Delta XYZ}$

$\therefore \frac{4.5}{XY} = \frac{6}{YZ} = \frac{8}{XZ} = \frac{18.5}{74}$

$\therefore$ The longest side is $\overline{XZ}$

, $XZ = 32$ cm. (The req.)

22

$\because \Delta AED \sim \Delta ADB$

$\therefore m(\angle ADE) = m(\angle ABD)$

$\therefore X + 20° = 2X + 5° \qquad \therefore X = 15°$

$\therefore m(\angle ADE) = 15° + 20° = 35°$ (the req.)

23

$\because \overline{XY} \parallel \overline{BN}$, $\overleftrightarrow{YZ}$ is a transversal to them.

$\therefore m(\angle Y) = m(\angle NBZ)$ (corresponding angles) (1)

$\because \overline{XY} \parallel \overrightarrow{BN}$, $\overleftrightarrow{XZ}$ is a transversal to them.

$\therefore m(\angle X) = m(\angle BNZ)$ (corresponding angles) (2)

In $\Delta\Delta$ XYZ , NBZ :

$\because \angle Z$ is a common angle.

From (1) and (2) :

$\therefore \Delta XYZ \sim \Delta NBZ$ (4)

Similarly we can prove that :

$\Delta ABC \sim \Delta NBZ$ (5)

From (4) and (5)

$\therefore \Delta XYZ \sim \Delta NBZ \sim \Delta ABC$ (the first req.)

From (5) we find that : $\frac{BZ}{BC} = \frac{NZ}{AC}$

$\therefore \frac{4}{BC} = \frac{6}{12} \qquad \therefore BC = 8$ cm.

$\therefore$ Z is the midpoint of $\overline{BC}$ (the second req.)

Then ZC = 4 cm.

From (4) we find that : $\frac{XZ}{NZ} = \frac{YZ}{BZ}$

$\therefore \frac{8}{6} = \frac{YZ}{4} \qquad \therefore YZ = \frac{32}{6} = 5\frac{1}{3}$ cm.

$\therefore YC = YZ + ZC = 5\frac{1}{3} + 4 = 9\frac{1}{3}$ cm. (the third req.)

24

$\because \overline{XY} \parallel \overline{AB}$, $\overline{AX} \parallel \overline{BY}$

$\therefore$ ABYX is a parallelogram.

$\because m(\angle B) = 90° \qquad \therefore$ ABYX is a rectangle

$\therefore BY = AX = 4$ cm.

$\because BC = AD = 12$ cm. $\qquad \therefore YC = 12 - 4 = 8$ cm.

$\because \Delta AXM$ is right-angled at X

$\therefore (AM)^2 = (AX)^2 + (XM)^2 = 16 + 9 = 25$

$\therefore AM = 5$ cm.

In $\Delta\Delta$ AMX , CMY :

$m(\angle AXM) = m(\angle MYC) = 90°$

$m(\angle AMX) = m(\angle CMY)$ (V.O.A)

$\therefore m(\angle XAM) = m(\angle MCY)$

$\therefore \Delta AMX \sim \Delta CMY$ (the first req.)

$\therefore \frac{AX}{CY} = \frac{XM}{YM} = \frac{AM}{CM} \qquad \therefore \frac{4}{8} = \frac{3}{YM} = \frac{5}{CM}$

$\therefore YM = \frac{3 \times 8}{4} = 6$ cm $\qquad CM = \frac{5 \times 8}{4} = 10$ cm.

$\therefore$ The perimeter of $\Delta CYM = 6 + 10 + 8 = 24$ cm.

(the second req.)

$\because AB = CD \qquad \therefore \frac{AB}{CD} = 1$

$\because \frac{YM}{XM} = \frac{6}{3} = 2 \qquad \therefore \frac{AB}{CD} \neq \frac{YM}{XM}$

$\therefore$ The figure ABYM is not similar to the figure CDXM

(the third req.)

25

$\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{EC}$ is a transversal to them.

$\therefore m(\angle C) = m(\angle AED)$ (corresponding angles)

Similarly we can prove that :

$m(\angle B) = m(\angle ADE)$ (corresponding angles)

In $\Delta\Delta$ ADE , ABC :

$\angle A$ is a common angle, $m(\angle C) = m(\angle AED)$,

$m(\angle B) = m(\angle ADE)$

$\therefore \Delta ABC \sim \Delta ADE$

$\therefore \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$

$\therefore \frac{3}{(X+1)+3} = \frac{2}{(X-1)+2} = \frac{y}{12}$ (1)

$\therefore \frac{3}{X+4} = \frac{2}{X+1} = \frac{y}{12} \qquad \therefore 3(X+1) = 2(X+4)$

$\therefore 3X + 3 = 2X + 8$

$\therefore X = 5 \qquad \therefore AB = 9$ cm. (the first req.)

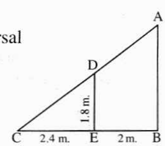
EC = 4 cm. (the second req.)

Substituting in (1) :

$\therefore \frac{3}{9} = \frac{y}{12} \qquad \therefore y = \frac{3 \times 12}{9} = 4$ cm.

$\therefore DE = 4$ cm. (the third req.)

26



$\because \overline{DE} \parallel \overline{AB}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle A) = m(\angle CDE)$

(corresponding angles)

, $\because \overline{DE} \parallel \overline{AB}$, $\overleftrightarrow{BC}$

is a transversal

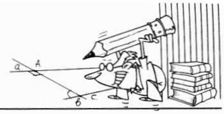
$\therefore m(\angle B) = m(\angle CED)$ (corresponding angles)

, $\because \angle C$ is common

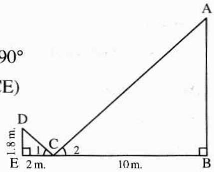
$\therefore \Delta ABC \sim \Delta DEC \qquad \therefore \frac{AB}{DE} = \frac{BC}{EC} = \frac{AC}{DC}$

$\therefore \frac{AB}{1.8} = \frac{4.4}{2.4}$

$\therefore AB = \frac{1.8 \times 4.4}{2.4} = 3.3$ m. (The req.)



27



In $\Delta\Delta$ ABC , DEC :

$\because$ m ($\angle$ B) = m ($\angle$ E) = 90°

, m ($\angle$ ACB) = m ($\angle$ DCE)

$\therefore$ m ($\angle$ A) = m ($\angle$ D)

$\therefore \Delta ABC \sim \Delta DEC$

$\therefore \frac{AB}{DE} = \frac{BC}{CE}$

$\therefore \frac{AB}{1.8} = \frac{10}{2}$ $\therefore AB = \frac{1.8 \times 10}{2} = 9$ m. (The req.)

28

$\because \frac{AB}{BC} = \frac{DE}{EF}$ $\therefore \frac{AB}{DE} = \frac{BC}{EF}$

$\because$ AB = AC , DE = DF $\therefore \frac{AC}{DF} = \frac{AB}{DE} = \frac{BC}{EF}$

$\therefore \Delta ABC \sim \Delta DEF$ (Q.E.D.)

29

In Δ ABC :

$\because x + 2x + 3x = 180°$

$\therefore 6x = 180°$ $\therefore x = 30°$

$\therefore$ m ($\angle$ C) = 30° , m ($\angle$ B) = 60° , m ($\angle$ A) = 90°

$\therefore AB = \frac{1}{2}$ BC

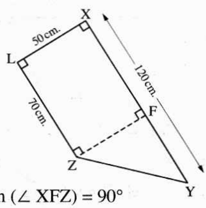
$\therefore AB = \frac{1}{2} \times 18 = 9$ cm.

$\because \Delta ABC \sim \Delta DEF$

$\therefore \frac{AB}{DE} = \frac{BC}{EF}$ $\therefore \frac{9}{3} = \frac{18}{EF}$

$\therefore$ EF = 6 cm. (Q.E.D.)

30



Constr. :

Draw :

$\overline{CE} \perp \overline{AB}$, $\overline{FZ} \perp \overline{XY}$

Proof :

In the quadrilateral XLZF

$\because$ m ($\angle$ X) = m ($\angle$ L) = m ($\angle$ XFZ) = 90°

$\therefore$ m ($\angle$ LZF) = 90°

$\therefore$ The figure XLZF is a rectangle.

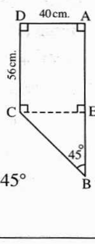
$\therefore$ FZ = XL = 50 , XF = LZ = 70 cm.

$\therefore$ FY = XY – XF = 120 – 70 = 50 cm.

In Δ ZFY which is right-angled at F :

$\because$ FY = FZ

$\therefore$ m ($\angle$ Y) = m ($\angle$ FZY) = $\frac{180° - 90°}{2}$ = 45°



$\because (YZ)^2 = (FY)^2 + (FZ)^2$

$\therefore (YZ)^2 = 2500 + 2500 = 5000$

$\therefore YZ = 50\sqrt{2}$ cm.

In the quadrilateral ADCE :

$\because$ m ($\angle$ A) = m ($\angle$ D) = m ($\angle$ AEC) = 90°

$\therefore$ m ($\angle$ DCE) = 90°

$\therefore$ The figure ADCE is a rectangle.

$\therefore$ EC = AD = 40 cm. , AE = DC = 56 cm.

In Δ BEC which is right-angled at E :

$\because$ m ($\angle$ B) = 45°

$\therefore$ m ($\angle$ ECB) = 180° – (90° + 45°) = 45°

$\therefore$ m ($\angle$ B) = m ($\angle$ ECB) $\therefore$ EB = EC = 40 cm.

$\because (BC)^2 = (BE)^2 + (EC)^2$

$\therefore (BC)^2 = 1600 + 1600 = 3200$

$\therefore BC = 40\sqrt{2}$ cm.

$\because$ AB = AE + EB $\therefore$ AB = 56 + 40 = 96 cm.

In the two figures ABCD , XYZL :

$\because$ m ($\angle$ A) = m ($\angle$ X) = 90° , m ($\angle$ B) = m ($\angle$ Y) = 45°

m ($\angle$ C) = m ($\angle$ Z) = 135°

m ($\angle$ D) = m ($\angle$ L) = 90°

$\frac{AB}{XY} = \frac{96}{120} = \frac{4}{5}$, $\frac{BC}{YZ} = \frac{40\sqrt{2}}{50\sqrt{2}} = \frac{4}{5}$

$\frac{DC}{LZ} = \frac{56}{70} = \frac{4}{5}$, $\frac{AD}{LX} = \frac{40}{50} = \frac{4}{5}$

$\therefore \frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{LZ} = \frac{DA}{LX} = \frac{4}{5}$

$\therefore$ The figure ABCD $\sim$ the figure XYZL (The req.)

Answers of Exercise 7

1

Fig. (1) :

$\because (AB)^2 = 25$, $(BC)^2 = 144$, $(AC)^2 = 169$

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$\therefore$ m ($\angle$ B) = 90°

Fig. (2) :

$\because (AB)^2 = 225$, $(BC)^2 = 400$, $(AC)^2 = 625$

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$\therefore$ m ($\angle$ B) = 90°

Fig. (3) :

$\because (AB)^2 = 324$, $(BC)^2 = 576$, $(AC)^2 = 900$

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$\therefore$ m ($\angle$ B) = 90°